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Decision Support

Product differentiation and entry timing in a continuous time spatial competition model[☆]

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ABSTRACT

We extend the well-known spatial competition model (d'Aspremont, Gabszewicz & Thisse, 1979) to a continuous time model in which two firms compete in each instance. Our focus is on the entry timing decisions of firms and their optimal locations. We demonstrate that the leader has an incentive to locate closer to the center to delay the follower's entry, leading to a non-maximum differentiation outcome. We also investigate how exogenous parameters affect the leader's location and firms' values and, in particular, numerically show that the profit of the leader changes non-monotonically with an increase in the transport cost parameter.

1. Introduction

Researchers in economics and marketing have emphasized the importance of (horizontal) product differentiation in the context of firm competition (e.g. Brown, 1989; d'Aspremont, Gabszewicz, & Thisse, 1979; Lancaster, 1990). When firms launch their new products into markets, timing and product characteristics are some of the important factors for their profits (e.g. Krishnan & Ulrich, 2001). Taking into account firms' decisions regarding product differentiation, researchers theoretically and/or empirically investigate how firms determine the timing of launching their products and those characteristics (e.g. Lambertini, 1997; Thomadsen, 2007).

From the theoretical point of view, Lambertini (2002) presented pioneering work that discusses the strategic interaction between the optimal locations of the inventor (the market leader), who anticipates subsequent entry and the location choice of the follower in a Hoteling-type spatial competition model, as in d'Aspremont et al. (1979).[3] He was the first to introduce a dynamic model in the sense that time is continuous, the firm locations are fixed once entry is made and that firms earn their profits in each instance. Regarding the time structure, several papers deal with sequential locational entry in a discrete time model, which allows qualitative analyses such as how many steps the timing of investment would change given a change in other parameters (e.g. Prescott & Visscher, 1977). However, a more rigorous quantitative analysis, such as determining the percentage change in the investment time attributable to a percentage change in a parameter, requires a continuous time model.[4]

This novel point is from Lambertini (2002) and differs significantly from those in related theoretical papers discussing sequential

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[3] Location point is interpreted as a firm's differentiation selection because the distance between a firm's location point and a consumer's address corresponds to that between a firm's attribute and a consumer's ideal point. This interpretation is standard in spatial economics and marketing literature.

[4] Continuous time models are often used in models such as real option game models that investigate the timing problem of firms' entry without the locational context (e.g. Dixit & Pindyck, 1994, Chapter 9; Azevedo & Paxson, 2014). These studies introduce one or more probabilistic fluctuations into their models. Our model is deterministic and does not focus on this randomness, but instead, focuses on the relation between location and entry timing. The real option game approach is useful for taking into account the endogenous timing decision.

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location choices based on Hoteling-type spatial competition models (e.g. Götz, 2005; Neven, 1987).[5] Those related papers are static Hoteling models in the sense that each firm has only one profit earning chance.[6] Lambertini (2002) considered two scenarios: (i) the follower's timing of entry is exogenous and (ii) the follower's timing of entry is probabilistically determined. Therefore, the follower does not endogenously determine its optimal timing of entry in either scenario. To summarize, Lambertini (2002) considered a continuous *time* model, but an endogenous entry *timing* model with continuous time *has not been considered* in locational models. Because the entry timing of followers significantly influences market leaders as well as followers (Kalyanaram, Robinson, & Urban, 1995; Vakratsas, Rao, & Kalyanaram, 2003), we need to overcome the weakness in the model given by Lambertini (2002) and endogenize the follower's entry-timing decision. Therefore, our paper substantially extends the model of Lambertini (2002).

We incorporate several aspects into the standard Hoteling duopoly model in d'Aspremont et al. (1979). The time horizon is infinite, as in Lambertini (2002). Each firm sets a price and earns a profit in each instance if it exists in the market, implying that a delay of entry causes a loss of profit opportunity. In anticipation of subsequent entry by the follower, the market leader initially sets its location. Because the leader's location decision influences the profits of the follower, it also affects the timing of the entry (the length of the monopoly period), thus representing an additional value of our paper. After the location choice of the market leader, the follower determines the timing of entry and its location. When the follower enters the market, it incurs an investment cost that exponentially decreases with the standard discount rate. In contrast, consumer size increases with a growth rate lower than the discount rate. By balancing the benefit and cost of staying outside, the follower determines its entry timing and location. We also note that this formulation is suitable for perishable goods as consumers repeatedly purchase the good.[7]

Compared with Lambertini (2002), our contributions in this paper are threefold. The first contribution is that we endogenize the follower's timing. The second contribution follows the first, as we introduce investment costs and a growth rate in consumer size to make the model more realistic. In addition, the growth rate ensures that the entry occurs within a finite time[8] and, in turn, affects the leader's location. The third contribution is a strategic interaction between the leader's location and the follower's entry timing. In addition to the effects considered by Lambertini (2002), the leader's moving closer to the center increases the follower's incentive to delay its entry, prolonging the monopoly regime. Thus, by endogenizing the follower's timing, the leader has a stronger incentive to move closer to the central point. Although this strategic interaction among the leader's location, the follower's location and its entry timing is an important aspect of this problem, Lambertini (2002) does not take into account this strategic interaction because of his assumption of an exogenous entry timing by the follower.

We also show that the follower always chooses to maximize the distance between the firms whereas the leader has an incentive to locate closer to the center to delay the follower's entry, possibly leading to a non-maximum differentiation outcome. Furthermore, the location interval between the leader and the follower is negatively correlated with the length of time for which the follower stays outside. These results are similar to those in Lambertini (2002), although the mechanism behind these results definitely differs between the two papers.

Finally, we show that the entry timing becomes earlier as the growth rate of consumer size or the parameter of consumer transport cost increases, and becomes later as the discount rate increases. We numerically investigate how those exogenous parameters influence their profits. A notable result is that the profit of the market leader non-monotonically changes with an increase in the consumer transport cost parameter.[9]

2. The model

Two firms, $i \in \{1, 2\}$, produce homogeneous goods. Consumers are uniformly distributed over the unit segment $[0, 1]$ as proposed by Hoteling (1929).[10] Each consumer at point $x \in [0, 1]$ repeatedly purchases at each instance $[t, t + dt)$ at most one unit of the good and decides from which firm to purchase if he does make a purchase.[11] The consumer at point $x \in [0, 1]$ incurs a quadratic transportation cost $c(x_i - x)^2$ and pays price p_{it} at time $t \in [0, \infty)$ when buying a good from firm i located at $x_i \in [0, 1]$. To summarize, the utility of the consumer at point $x \in [0, 1]$ at time $t \in [0, \infty)$ is given by

$$u_t(x; x_1, x_2, p_{1t}, p_{2t}) = \begin{cases} \bar{u} - p_{1t} - c(x_1 - x)^2 & \text{if purchased from firm 1,} \\ \bar{u} - p_{2t} - c(x_2 - x)^2 & \text{if purchased from firm 2,} \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where $\bar{u}$ denotes the gross surplus that a consumer at point x enjoys from purchasing the good, and c is a parameter describing the level of transportation cost or product differentiation. Let us assume that $\bar{u}$ is so large that each consumer prefers to purchase one good over not buying when at least one firm is present in the market.[12]

Assumption 1. $\bar{u} > 3c$.

The game proceeds as follows: each firm i chooses the time of entry $T_i \in [0, \infty)$ and location $x_i \in [0, 1]$ at the same time, and then chooses price $p_{it} : \Re_+ \to \Re_+$ at each time t, which is a function from time $t \in [0, \infty)$ to a real number $[0, \infty)$ and is displayed as p_{it} for simplicity. In addition, we assume that firm 1 is the leader who just entered at $T_1 = 0$, whereas firm 2 is the follower who enters at time T_2, to be subsequently and endogenously determined. In this way, firm 1 decides x_1 at time $T_1 = 0$ once and subsequently chooses price p_{1t} at each time t. After observing firm 1's actions before firm 2's entry, firm 2 chooses to enter at time T_2 and location x_2 and thereafter chooses p_{2t} at each time t. Firm i can choose its location only when it makes its entry in the project, at which time it incurs an entry cost $F_i(T_i)$. We also assume (without loss of generality) that $x_1 \leq 1/2$ holds in equilibrium.

Now, let us describe the present value of the firms at time 0 given that firm 2 would enter at point x_2 at time $t = T_2$. Note that firm 1 enters at point x_1 at time $t = T_1 = 0$. The timing is exogenous[13] but x_1

[5] Many papers discussed sequential location choices in spatial competition models. Kress and Pesch (2012) and Biscaia and Mota (2013) provided comprehensive surveys on spatial competition.

[6] Lambertini (1997) and Meza and Tombak (2009) considered the endogenous timing of locations in such static Hoteling models.

[7] Perishable goods are defined as non-durable goods that last only for each infinitesimal instance of time. We will mention this point further in Discussion and concluding remarks.

[8] This phenomenon implies that just introducing a timing endogeneity into Lambertini (2002) without a growth rate yields no entry and a perpetual monopoly by the leader.

[9] The transport cost parameter can be interpreted as a parameter that describes the level of product differentiation because the cost parameter corresponds to a consumer's disutility between the consumer's ideal point and the degree of a product's attribute. This interpretation is standard in the literature on spatial economics and marketing.

[10] This setting and the following assumptions are standard in the literature on spatial economics.

[11] All consumers will respectively purchase a unit of product in equilibrium due to Assumption 1.

[12] In other words, firm 1, located at $x_1 = 0$, has an incentive to supply a positive amount at location 1, after maximizing its profit.

[13] A similar interpretation is made in Chronopoulos, De Reyck, and Siddiqui (2014) as a non-preemptive duopoly. In their paper, the roles of the leader and the follower are defined exogenously. Consequently, the future cash flows of the leader are discounted to time $t = 0$.

is to be determined endogenously in the following analysis. The total profit of firm 1, the leader, is given by

$$V_1(T_2, x_1, x_2, p_{1t}, p_{2t}) = \int_0^{T_2} \int_0^1 p_{1t}(x; x_1) e^{-(r-\alpha)t} dxdt + \int_{T_2}^{\infty} \int_0^{\bar{x}} p_{1t}(x; x_1, x_2, p_{2t}) e^{-(r-\alpha)t} dxdt - F_1(0), \quad (2)$$

where r denotes the interest rate, α denotes the growth rate parameter of the market and $\bar{x}$ denotes a point at which the consumer is indifferent between purchasing from firm 1 or 2.[14] From Eq. (1), $\bar{x} = [p_{2t} - p_{1t} + c(x_2^2 - x_1^2)]/[2c(x_2 - x_1)]$. We assume that $r > \alpha$ to ensure that the follower enters in a finite time.[15] Firm 1 earns monopoly profit flow at $t \in [\tau, \tau + d\tau)$ until firm 2 enters the market, and firm 1 earns duopoly profit flow after firm 2's entry.

The total profit of firm 2, the follower, is given by

$$V_2(T_2, x_1, x_2, p_{1t}, p_{2t}) = \int_{T_2}^{\infty} \int_{\bar{x}}^1 p_{2t}(x; x_1, x_2, p_{1t}) e^{-(r-\alpha)t} dxdt - F_2(T_2). \quad (3)$$

After entry, firm 2 earns duopoly profit flow at $t \in [\tau, \tau + d\tau)$ to which it supplies.

Let us now make the following assumptions for the entry cost function F_i.

Assumption 2.

(i) $F_i(T_i) = F_i e^{-rT_i}$,
(ii) $F_1(0) = F_1 < \int_0^{\infty} \int_0^{\bar{x}} p_{1t}(x; x_1, x_2, p_{2t}) e^{-(r-\alpha)t} dxdt$ for all $x_2 \in [x_1, 1]$,
(iii) $F_2(0) = F_2 > \int_0^{\infty} \int_{\bar{x}}^1 p_{2t}(x; x_1, x_2, p_{1t}) e^{-rt} dxdt$ for all $x_1 \in [0, x_2]$.

Assumption 2 (ii) suggests that F_1 is small enough that the leader can earn non-negative total profit at time 0 whenever the follower enters and wherever the follower locates. Similarly, Assumption 2(iii) suggests that F_2 is sufficiently large enough so that it is optimal for the follower to enter sequentially at $T_2 > 0$ in an equilibrium, because we would like to avoid simultaneous entry at time 0 and focus on sequential entry.[16] This assumption also implies that the leader, but not the follower, has already learned how to enter the market efficiently.

3. Equilibrium

In this section, we derive the price, location and timing outcomes in the subgame perfect equilibrium. First, given locations x_1 and x_2, we consider the problem of prices at each time t before and after the entry of firm 2. Then, we derive the local profits of the leader and the follower at each time t.

The following are the equilibrium prices. Notably, the maximization of the instantaneous profit flows is equivalent to the maximization of the total profits. In other words, firm 1 maximizes the following equation with respect to p_{1t} before firm 2 enters at $t \in [0, T_2)$:

$$\max_{p_{1t}} \int_0^1 p_{1t}(x; x_1) dx. \quad (4)$$

[14] We can interpret α as the increasing rate of consumer population. Let N denote the population of consumers at time 0. Then, the population at time t is $Ne^{\alpha t}$. In our model, we normalize $N = 1$. Therefore, our interpretation of α being the market growth rate is equivalent to this population interpretation. It is typical to consider population in the standard Hoteling setting. Since Malthus (1798)'s famous argument, academic scholars often set assumptions that the sizes of population and/or market grow exponentially. The validity of exponential market growth is noted in many industries, for example, Lages and Fernandes (2005) on telecommunication services, Victor and Ausubel (2002) on DRAM, and Vakratsas and Kolsarici (2008) on pharmaceuticals.

[15] If $r \le \alpha$, the integral of Eq. (3) could be made indefinitely larger by choosing a larger time T_2. Thus, waiting longer would always be a better strategy, and the optimum would not exist.

[16] If simultaneous entry occurs, this model reverts to the standard location–price model of d'Aspremont et al. (1979).

After firm 2 enters at $t \in [T_2, \infty)$, firm 1 maximizes the following with respect to p_{1t}:

$$\max_{p_{1t}} \int_0^{\bar{x}} p_{1t}(x; x_1, x_2, p_{2t}) dx; \quad (5)$$

and firm 2 maximizes the following with respect to p_{2t}:

$$\max_{p_{2t}} \int_{\bar{x}}^1 p_{2t}(x; x_1, x_2, p_{1t}) dx. \quad (6)$$

Solving these maximization problems results in the following lemma.

Lemma 1. *The prices set by the leader and the follower are*

$$\tilde{p}_{1t} = \begin{cases} p_1^M = \bar{u} - c(1 - x_1)^2 & t \in [0, T_2) \\ p_1^D = \frac{c}{3}(x_2 - x_1)(2 + x_1 + x_2) & t \in [T_2, \infty), \end{cases} \quad (7)$$

$$\tilde{p}_{2t} = p_2^D = \frac{c}{3}(x_2 - x_1)(4 - x_1 - x_2) \qquad t \in [T_2, \infty). \quad (8)$$

Proof. See Appendix □

For $t \in [0, T_2)$, the monopoly leader maximizes its price under the constraint that all consumers purchase its good. Assuming that $x_1 \le 1/2$ without loss of generality, the furthest consumer is located at 1. The consumer turns out to be indifferent between purchasing the good at price p_1^M and not purchasing the good. p_i^D $(i = 1, 2)$ is derived using the standard calculation in the context of spatial competition (e.g. d'Aspremont et al., 1979).

The instantaneous profit flows of the two firms are

$$\pi_{1t}(x_1, x_2) = \begin{cases} \pi_1^M(x_1) = \int_0^1 p_1^M dx = \bar{u} - c(1 - x_1)^2 & t \in [0, T_2) \\ \pi_1^D(x_1, x_2) = \int_0^{\bar{x}} p_1^D dx = p_1^D \bar{x} \\ \quad = \frac{c}{18}(x_2 - x_1)(2 + x_1 + x_2)^2 & t \in [T_2, \infty), \end{cases} \quad (9)$$

$$\pi_{2t}(x_1, x_2) = \begin{cases} 0 & t \in [0, T_2) \\ \pi_2^D(x_1, x_2) = \int_{\bar{x}}^1 p_2^D dx = p_2^D(1 - \bar{x}) \\ \quad = \frac{c}{18}(x_2 - x_1)(4 - x_1 - x_2)^2 & t \in [T_2, \infty). \end{cases} \quad (10)$$

Note that $\bar{x} = (2 + x_1 + x_2)/6$. Substituting the outcomes of Eqs. (7)–(10) into Eqs. (2) and (3), the total profits of the leader and the follower are derived as

$$V_1(T_2, x_1, x_2, p_{1t}, p_{2t}) = \int_0^{T_2} \pi_1^M(x_1) e^{-(r-\alpha)t} dt + \int_{T_2}^{\infty} \pi_1^D(x_1, x_2) e^{-(r-\alpha)t} dt - F_1, \quad (11)$$

$$V_2(T_2, x_1, x_2, p_{1t}, p_{2t}) = \int_{T_2}^{\infty} \pi_2^D(x_1, x_2) e^{-(r-\alpha)t} dt - F_2(T_2). \quad (12)$$

3.1. Follower

We consider the problem of the follower regarding when it enters and where it locates in the market. With regards to the location, we have the following:

Lemma 2. *The follower always locates at* $x_2 = 1$.

Proof. From Eqs. (10) and (12), differentiating V_2 with respect to x_2, we have

$$\frac{\partial V_2(T_2, x_1, x_2, p_{1t}, p_{2t})}{\partial x_2} = \frac{e^{-(r-\alpha)T_2}}{r - \alpha} \frac{c}{18}(4 + x_1 - 3x_2)(4 - x_1 - x_2), \quad (13)$$

which is positive for x_1 and $x_2 \in [0, 1]$ when $r - \alpha > 0$ and $c > 0$. Thus, the optimal location for firm 2 is at 1. □

From Lemma 2, we show that the follower always locates as far away from the location of the leader as possible when entering the market. This result replicates that of Lambertini (2002) and seems to be robust to the endogeneity of the follower's entry timing. Substituting the equilibrium profits and the location of the follower into the total profit functions V_1 and V_2, we have

$$V_1(T_2, x_1, 1, \tilde{p}_{1t}, \tilde{p}_{2t}) = \frac{1 - e^{-(r-\alpha)T_2}}{r - \alpha}[\bar{u} - c(1 - x_1)^2] + \frac{e^{-(r-\alpha)T_2}}{r - \alpha}\frac{c(1 - x_1)(3 + x_1)^2}{18} - F_1, \quad (14)$$

$$V_2(T_2, x_1, 1, \tilde{p}_{1t}, \tilde{p}_{2t}) = \frac{e^{-(r-\alpha)T_2}}{r - \alpha}\frac{c(1 - x_1)(3 - x_1)^2}{18} - F_2 e^{-rT_2}. \quad (15)$$

We note that if $F_1 \le F_2$, the leader's value is always greater than the follower's value in our setting. That is, since $\pi_1^D \ge \pi_2^D$, we have

$$V_1 - V_2 \ge \frac{\pi_1^M}{r - \alpha}\left(1 - e^{-(r-\alpha)T_2}\right) - F_1(1 - e^{-rT_2}) > \left(\frac{\pi_1^M}{r - \alpha} - F_1\right)\left(1 - e^{-(r-\alpha)T_2}\right) > 0. \quad (16)$$

We use Assumption 2(ii) from the end of the last section to ensure $F_1 < \pi_1^M/(r - \alpha)$, the present value of profits when the leader sustains monopoly profit forever.

Thus, the rest of the follower's decision problem is only its endogenously determined entry timing, which is not present in Lambertini (2002). We have the following proposition.

Proposition 1. *The entry timing of the follower is*

$$\tilde{T}_2(x_1) = \frac{1}{\alpha}\log\left[\frac{rF_2}{\pi_2^D(x_1, 1)}\right] = \frac{1}{\alpha}\log\left[\frac{18rF_2}{c(1 - x_1)(3 - x_1)^2}\right]. \quad (17)$$

Proof. Differentiating $V_2(T_2, x_1, 1, \tilde{p}_{1t}, \tilde{p}_{2t})$ with respect to T_2 yields

$$\frac{\partial V_2(T_2, x_1, 1, \tilde{p}_{1t}, \tilde{p}_{2t})}{\partial T_2} = -e^{-(r-\alpha)T_2}\pi_2^D(x_1, 1) + rF_2 e^{-rT_2} = 0. \quad (18)$$

Solving this equation with respect to T_2 gives $\tilde{T}_2(x_1)$ as a solution. Note that $\tilde{T}_2$ is positive from Assumption 2(iii), which has

$$F_2 > \int_0^\infty \pi_2^D(x_1, x_2)e^{-rt}dt = \frac{\pi_{2t}^D(x_1, x_2)}{r}.$$

Examining the second-order derivative yields the following

$$\frac{\partial^2 V_2}{\partial T_2^2} = (r - \alpha)e^{-(r-\alpha)T_2}\pi_2^D(x_1, 1) - r^2 F_2 e^{-rT_2}. \quad (19)$$

Substituting $\tilde{T}_2(x_1)$ into the second-order derivative shows that $\partial^2 V_2/\partial T_2^2|_{T_2=\tilde{T}_2(x_1)}$ is negative. Because the first-order condition is uniquely satisfied and the second-order derivative is negative at this point, the unique, positive and interior solution exists. Thus, we have the desired result. □

The following corollaries show the change in the follower's entry timing as the leader's location x_1 and the exogenous parameters change.

Corollary 1. *If x_1 is increased, the optimal timing for the follower to enter is delayed.*

x_1 affects the entry timing of the follower as follows. Only the denominator within the fraction inside the log in Eq. (17) is composed of the locations chosen by the firms. Thus, we focus on this part of the equation, namely, $\pi_2^D(x_1, 1)$. When firm 1 locates away from firm 2 (or x_1 is decreased), firm 2 can deliver the product in a broader region ($\bar{x}$ decreases), and firm 2 can earn a higher profit (π_2^D increases). Therefore, firm 2 enters earlier (later) if firm 1 locates away from (closer to) firm 2.

Corollary 2. *If F_2 or r is increased, or as c or α is decreased, the optimal timing for the follower to enter is delayed.*

As is seen in the next subsection, x_1 depends on the previous parameters. Therefore, a change in these parameters affects the entry timing of the follower through a direct effect (Corollary 2) and an indirect effect (Corollary 1).

3.2. Leader

Finally, we consider the problem of the leader. Substituting the outcomes of the follower, Lemma 2 and Proposition 1 into the total profit function of the leader, the maximization problem of the leader is given by

$$\max_{x_1 \in [0, 1/2]} V_1(\tilde{T}_2(x_1), x_1, 1, \tilde{p}_{1t}, \tilde{p}_{2t}) = \frac{1 - e^{-(r-\alpha)\tilde{T}_2(x_1)}}{r - \alpha}[\bar{u} - c(1 - x_1)^2] + \frac{e^{-(r-\alpha)\tilde{T}_2(x_1)}}{r - \alpha}\frac{c(1 - x_1)(2 + x_1 + 1)^2}{18} - F_1. \quad (20)$$

Because the derivation of the equilibrium is complicated, we investigate the impact on the value of the leader when the location of the leader changes infinitesimally, before deriving the equilibrium outcome presented in Proposition 3. Differentiating $V_1(\tilde{T}_2(x_1), x_1, 1, \tilde{p}_{1t}, \tilde{p}_{2t})$ with respect to x_1 yields

$$\frac{\partial V_1(\tilde{T}_2(x_1), x_1, 1, \tilde{p}_{1t}, \tilde{p}_{2t})}{\partial x_1} = e^{-(r-\alpha)\tilde{T}_2(x_1)}\frac{d\tilde{T}_2(x_1)}{dx_1} \times (\pi_1^M(x_1) - \pi_1^D(x_1, 1)) + \frac{1 - \exp(-(r - \alpha)\tilde{T}_2(x_1))}{r - \alpha}\frac{d\pi_1^M(x_1)}{dx_1} + \frac{\exp(-(r - \alpha)\tilde{T}_2(x_1))}{r - \alpha}\frac{d\pi_1^D(x_1, 1)}{dx_1}. \quad (21)$$

The sign of Eq. (21) is the key and determines the location of the leader: whether it is located at the center (1/2), edge (0), or an interior point (strictly between 0 and 1/2). The first term of Eq. (21) represents the gain from the delay of entry by firm 2 that is caused by an increase in x_1, allowing firm 1 to maintain its monopoly profit before the duopoly regime begins. This term is not present in Lambertini (2002) and captures the essence behind our results. Therefore, if this term turns out to be large, our result may become significantly different from that of Lambertini (2002). The second term of Eq. (21) signifies the increase in the monopoly profit attributable to moving closer to the center. The third term of Eq. (21) shows how the duopoly profit decreases as firm 1 moves closer to firm 2, thus intensifying competition.[17] The first two terms are positive and the last term is negative. Thus, if the effect of the last term is relatively small, the optimal location of firm 1 is 1/2.

We now investigate the effect of each parameter on the equilibrium location of firm 1, which we denote as x_1^E. First, we offer a proposition regarding the effects of parameter α on x_1^E.

Proposition 2. *Regarding $\alpha \in (0, r)$: (a) If α is sufficiently large, $x_1^E = 0$. (b) If α is sufficiently small, $x_1^E = 1/2$. (c) There exists $\hat{\alpha} \in (0, r)$ such that the optimal location for firm 1 is interior (i.e. $x_1^E \in (0, 1/2)$).*

Proof. See Appendix □

[17] Although T_2 is not endogenized, the second and third terms describe how Lambertini (2002) determined the leader's location.

As α approaches r, the weight on the periods after which firm 2 enters the market gets larger. Therefore, firm 1 needs to take into account profitability under the duopoly situation. The significance of the last term dominates that of the other two terms, which implies that the optimal location of firm 1 is $x_1 = 0$. In contrast, if α becomes smaller, the converse holds. Since V_1 is continuous with respect to α, there always exists some level of α that leads to an interior solution for x_1. Meanwhile, the effect of parameter r is essentially the opposite of that of α.

Parameter $\bar{u}$ appears only in the first term, which is increasing in $\bar{u}$. As the location of firm 1 becomes closer to the center, the leader obtains the monopoly profit for a longer time because the follower's timing of entry is delayed from Corollary 1. An increase in $\bar{u}$ yields a stronger incentive to obtain this profit. Thus, as $\bar{u}$ increases, the leader is more likely to locate at $x_1^E = 1/2$.

Regarding F_2 and c, they affect Eq. (21) through $\tilde{T}_2$ in that an increase in F_2 or a decrease in c increases $\tilde{T}_2$. Thus, the effects of these two parameters counter each other. Note that the term $d\tilde{T}_2(x_1)/dx_1$ is independent of c or F_2. From Corollary 2, increasing F_2 delays the entry of the follower (increasing $\tilde{T}_2$), and F_2 does not affect Eq. (21) in other ways. The discounted present value of the increase in profit from a longer monopoly regime attributable to firm 1 locating closer to the center is decreased. Thus, the first term is decreasing in F_2. The second term is increasing in F_2 because the increase in the monopoly profit for firm 1 from its moving closer to the center is sustained longer due to the entry delay. Similarly, the third term is also increasing in F_2, as the decrease in the duopoly profit for firm 1 from its moving closer to the center is devalued from the entry delay. These three effects are complicated, and none of them are analytically dominant. We numerically examine this issue in the next section.

The effect of parameter c counters that of F_2 with respect to how they affect $\tilde{T}_2$. In addition, c enters in all the profit levels. Namely, c has two contrasting effects on the profit of firm 1. First, the monopoly profit of firm 1 is decreasing in c. When the firm supplies to all consumers, it needs to compensate consumers for transport costs by lowering its monopoly price. The compensation is higher as the consumer transport cost parameter increases. Second, the parameter c is positively related to the duopoly profit of firm 1 as in the standard Hoteling model with price competition. The effect of the first term is lower as the parameter c increases, whereas those of the second and third terms are higher. Therefore, the relative importance of the three terms influences the effect of c on the location choice of firm 1. Unlike the previous argument, this effect depends on the parameters α and r. We apply the previous argument regarding α to the effect of parameter c on the optimal location of firm 1. If α approaches r, the significance of the last term dominates that of the second term, which implies that the optimal location of firm 1 is more likely to become $x_1^E = 0$ as c increases. In contrast, if α becomes smaller, the significance of the second term dominates the last, which implies that the optimal location of firm 1 is more likely to become $x_1^E = 1/2$ as c increases. If r is sufficiently large, for example $r > 1 + \alpha$, the optimal location of firm 1 is more likely to become $x_1^E = 0$ as c increases because the first term is relatively large.

Finally, solving Eq. (20), we have the following proposition.

Proposition 3. *(a) If Eq. (21) is positive for any $x_1 \in [0, 1/2]$, the outcome of the subgame perfect equilibrium is*

$$T_2^* = \frac{1}{\alpha}\log\left[\frac{144rF_2}{25c}\right], \quad x_1^* = \frac{1}{2}, \quad x_2^* = 1, \quad \bar{x}^* = \frac{7}{12}, \tag{22}$$

$$p_{1t}^* = \begin{cases} \tilde{p}_1^M(x_1^*) = \bar{u} - \frac{c}{4} & t \in [0, T_2^*) \\ p_1^D(x_1^*, x_2^*) = \frac{7c}{12} & t \in [T_2^*, \infty), \end{cases}$$
$$p_{2t}^* = p_2^D(x_1^*, x_2^*) = \frac{5c}{12} \qquad t \in [T_2^*, \infty). \tag{23}$$

(b) If (21) is negative for any $x_1 \in [0, 1/2]$, the outcome of the subgame perfect equilibrium is

$$T_2^{**} = \frac{1}{\alpha}\log\left[\frac{2rF_2}{c}\right], \; x_1^{**} = 0, \; x_2^{**} = 1, \; \bar{x}^{**} = \frac{1}{2}, \tag{24}$$

$$p_{1t}^{**} = \begin{cases} p_1^M(x_1^{**}) = \bar{u} - c & t \in [0, T_2^{**}) \\ p_1^D(x_1^{**}, x_2^{**}) = c & t \in [T_2^{**}, \infty), \end{cases}$$
$$p_{2t}^{**} = p_2^D(x_1^{**}, x_2^{**}) = c \qquad t \in [T_2^{**}, \infty). \tag{25}$$

(c) If the equilibrium location of firm 1, x_1^{***}, is strictly between 0 and 1/2, then the outcome of subgame perfect equilibrium is

$$T_2^{***} = \frac{1}{\alpha}\log\left[\frac{18rF_2}{c(1-x_1^{***})(3-x_1^{***})^2}\right],$$
$$x_2^{***} = 1, \; \bar{x}^{***} = \frac{3+x_1^{***}}{6}, \tag{26}$$

$$p_{1t}^{***} = \begin{cases} p_1^M(x_1^{***}) = \bar{u} - c(1-x_1^{***})^2 & t \in [0, T_2^{***}) \\ p_1^D(x_1^{***}, x_2^{***}) = \frac{c}{3}(1-x_1^{***})(3+x_1^{***}) & t \in [T_2^{***}, \infty), \end{cases} \tag{27}$$

$$p_{2t}^{***} = p_2^D(x_1^{***}, x_2^{***}) = \frac{c}{3}(1-x_1^{***})(3-x_1^{***}) \qquad t \in [T_2^{***}, \infty). \tag{28}$$

Proposition 3 shows that three types of equilibrium location for the leader can emerge. Similar to Lambertini (2002), we show that firm 1 can locate at 0, 0.5 or an interior location depending on the parameter values. However, a case exists in which the introduction of endogenous entry timing leads to different equilibrium location outcomes compared with Lambertini (2002) using the same parameter values. To grasp the intuition behind this result, we proceed with a numerical analysis in the next section.

4. Numerical analysis

In this section, we investigate in detail the underlying properties of our model using numerical analysis. First, we investigate the effects of the key parameters, α, c and F_2, on firm 1's equilibrium location, which we denote as $x_1^E \in \{x_1^*, x_1^{**}, x_1^{***}\}$. In particular, focusing on the equilibrium-path behavior, we illustrate that all three types of equilibrium location patterns actually exist. Then, we show the importance of the endogeneity of T_2, namely the first term of Eq. (21). Finally, we examine the effects of the parameters on the total values of the firms in equilibrium.

4.1. The leader's location

This subsection examines the effects of parameters (α, c, F_2) on the equilibrium location of the leader.

First, let us consider the effect of α on x_1^E. Let us set the parameters as $r = 0.1$, $\bar{u} = 4$ and $c = 1$. We use three tables to illustrate the relationship between α and the equilibrium location for the leader. F_2 is set at 20, 50 and 100 for Tables 1–3, respectively.[18] The values of α are incremented by one ten-thousandth. As explained in Proposition 2, we pointed out that the location x_1^E moves from 1/2 to 0 as α approaches r. We confirm this result from all three tables.

Additionally, we investigate the effects of F_2 on x_1^E when α approaches r. $\overline{\alpha}$ denotes the lowest value for which the equilibrium x_1^E is 0, and $\underline{\alpha}$ denotes the highest value for which the equilibrium x_1^E is 1/2. Tables 1–3 imply that both $\overline{\alpha}$ and $\underline{\alpha}$ are increasing in F_2. This implication is consistent with the intuition offered in the previous

[18] We use these parameter values since they provide the best illustration for the interior (not 0 or 1/2) equilibrium location for firm 1.

Table 1
Location of firm 1, x_1^E, when $r = 0.1, \bar{u} = 4,\ F_2 = 20,$ and $c = 1$. We have $\underline{\alpha} = 0.0961,\ \overline{\alpha} = 0.0977$.

α	$\cdots$	0.0961	0.0962	$\cdots$	0.0969	$\cdots$	0.0976	0.0977	$\cdots$
x_1^E	0.5	0.5	0.432	$\cdots$	0.171	$\cdots$	0.014	0	0

Table 2
Location of firm 1, x_1^E, when $r = 0.1, \bar{u} = 4,\ F_2 = 50,$ and $c = 1$. We have $\underline{\alpha} = 0.0964,\ \overline{\alpha} = 0.0982$.

α	$\cdots$	0.0964	0.0965	$\cdots$	0.0973	$\cdots$	0.0981	0.0982	$\cdots$
x_1^E	0.5	0.5	0.453	$\cdots$	0.192	$\cdots$	0.011	0	0

Table 3
Location of firm 1, x_1^E, when $r = 0.1, \bar{u} = 4,\ F_2 = 100,$ and $c = 1$. We have $\underline{\alpha} = 0.0966,\ \overline{\alpha} = 0.0984$.

α	$\cdots$	0.0966	0.0967	$\cdots$	0.975	$\cdots$	0.0983	0.0984	$\cdots$
x_1^E	0.5	0.5	0.46	$\cdots$	0.212	$\cdots$	0.024	0	0

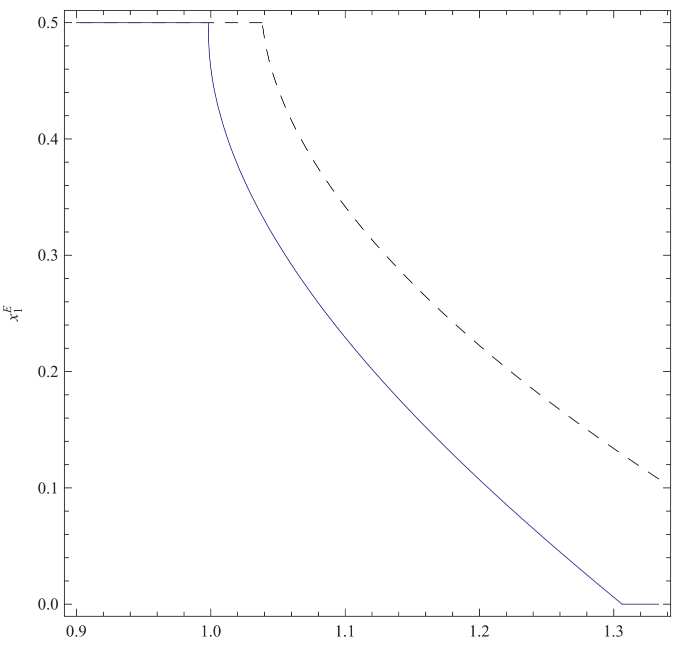

Fig. 1. Plotted values of optimal location for firm 1 with respect to $c \in (9/10, 4/3)$ when $F_2 = 20$ (dotted) and $F_2 = 13$ (solid).

section. In addition, the range of the interior location, $\overline{\alpha} - \underline{\alpha}$, is increasing in F_2. Therefore, as F_2 increases, the equilibrium location is likely to become interior or 1/2 and not 0.[19]

Second, we investigate the effect of c on the equilibrium location of the leader. Fig. 1 illustrates the equilibrium location of firm 1 for different values of c when $F_2 = 20$ and $F_2 = 13$. Consider the case in which an interior solution can exist. An interior solution exists when α approaches r. We consider the same values as in the previous numerical analysis; $r = 0.1,\ \alpha = 0.096$ and $\bar{u} = 4$. As $\bar{u} > 3c,\ c \in (0, 4/3)$. When $F_2 = 20$, the value of x_1^E declines from 1/2 to approximately 1/10 as c increases. In this case, firm 1 never locates at point 0. When $F_2 = 13$, x_1^E moves from 1/2 to 0 as c increases. Fig. 1 demonstrates the following concept. If the interior solution exists in the equilibrium, x_1^E is decreasing in c. As F_2 increases, x_1^E is more likely to take a higher value for a given c.

Third, we show the effects of both c and F_2 on the equilibrium location of the leader. Tables 4 and 5 illustrate the equilibrium location of firm 1 for different values of c and F_2 when $r = 0.1,\ \bar{u} = 4$ and $\alpha = 0.098$ for Table 4 and $\alpha = 0.097$ for Table 5. For these values of r and $\bar{u}$, these values of α allow interior solutions of x_1. Thus, for α very close to r, at approximately 0.099, we primarily have $x_1 = 0$; however, for most values of α, we tend to have the other corner solution at $x_1 = 1/2$.

By examining the interior solutions, we can better understand the effects of c and F_2. Values of c increase by 0.1 up to 1.3, as $\bar{u} > 3c,\ c \in (0, 4/3)$. F_2 is increased from 10 to 200 in increments of 10. Both tables show two trends. As c increases, if an interior solution exists in the equilibrium, x_1^E decreases. As F_2 increases, x_1^E increases if the interior solution exists in the equilibrium. As noted in the previous section, the effects of these two parameters tend to counter each other.

Finally, we have summarized how the parameters affect the equilibrium locations for the firms in Table 6. Note that this result is obtained under the parameter values in this section and depends on the magnitudes of parameters, in particular r and α.

4.2. On the importance of endogenous timing by the follower

One of the main points of this paper in contrast to previous papers, including Lambertini (2002), is that the follower's entry timing is endogenized and the leader determines its location considering this move. To determine its importance, we examine the relative sizes of the three terms in Eq. (21). Eq. (21) is the first-order derivative of V_1 with respect to x_1. The first term represents how the marginal change in the leader's location delays the follower's entry, allowing the leader to prolong its monopoly regime.

Using the parameters and the leader's location x_1 in Table 2, we derive concrete values of Eq. (21) in four cases. The common parameter values are $r = 0.1,\ \bar{u} = 4,\ c = 1$ and $F_2 = 50$. The result is summarized in Table 7.

The sum column in Table 7 indicates how firm 1 decides its location. If the value in the column is negative (positive), then firm 1 locates at 0 (0.5). If it is 0, an interior location between 0 and 0.5 may arise.[20]

The last four rows in Table 7 are interesting. If the first term is not present, the sum will be negative, and firm 1 has an incentive to move closer to 0. Therefore, compared with Lambertini (2002), firm 1 is more likely to locate closer to the center, forcing firm 2 to delay its entry. This effect of delaying entry does not exist in Lambertini (2002).

4.3. On total values of the firms

Finally, from the firms' point of view, their total values are more or at least as important as their entry timing and locations. We examine how the parameters $\bar{u}$ and c, commonly used in the Hoteling settings, affect firm values. The result for $\bar{u}$ is straightforward, but that for c is more complicated.

An increase in $\bar{u}$ improves the leader's value and possibly worsens the follower's value. $\bar{u}$ only appears in the monopoly profit phase of the leader. As it increases, firm 1 has more incentive to move closer to the center, giving firm 2 an incentive to delay its entry if the leader has not already located at the center. Because there is only one effect to be considered, the result is simple.

[19] These examples may illustrate that the range of α that leads to an interior location for firm 1 is rather small. However, our purpose in using these examples is to show that this range actually does exist and that the range of α that leads to locating away from 0, the edge of the market, is large.

[20] The first and third terms in Table 7 change in a non-monotonic manner with respect to α. This is because x_1^E is confined to be at 0 when the sum at the rightmost column is negative and at 0.5 when it is positive. When the sum is zero and x_1^E is strictly between 0 and 0.5, the terms move in a monotonic manner with respect to α.

Table 4
Location of firm 1, x_1^E, depending on the values of F_2 and c, when $r = 0.1, \alpha = 0.097, \bar{u} = 4$.

F_2/c	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3
200	0.5	0.5	0.5	0.5	0.5	0.5	0.418	0.353	0.304	0.265
190	0.5	0.5	0.5	0.5	0.5	0.5	0.414	0.348	0.299	0.260
180	0.5	0.5	0.5	0.5	0.5	0.5	0.408	0.343	0.294	0.255
170	0.5	0.5	0.5	0.5	0.5	0.5	0.403	0.338	0.288	0.249
160	0.5	0.5	0.5	0.5	0.5	0.5	0.397	0.332	0.283	0.243
150	0.5	0.5	0.5	0.5	0.5	0.497	0.391	0.323	0.276	0.236
140	0.5	0.5	0.5	0.5	0.5	0.489	0.384	0.319	0.269	0.229
130	0.5	0.5	0.5	0.5	0.5	0.480	0.377	0.312	0.262	0.222
120	0.5	0.5	0.5	0.5	0.5	0.470	0.369	0.303	0.254	0.213
110	0.5	0.5	0.5	0.5	0.5	0.460	0.360	0.295	0.245	0.204
100	0.5	0.5	0.5	0.5	0.5	0.448	0.350	0.285	0.234	0.193
90	0.5	0.5	0.5	0.5	0.5	0.436	0.339	0.273	0.223	0.181
80	0.5	0.5	0.5	0.5	0.5	0.422	0.326	0.261	0.210	0.167
70	0.5	0.5	0.5	0.5	0.5	0.406	0.311	0.246	0.194	0.151
60	0.5	0.5	0.5	0.5	0.5	0.387	0.294	0.228	0.175	0.132
50	0.5	0.5	0.5	0.5	0.5	0.365	0.272	0.205	0.152	0.107
40	0.5	0.5	0.5	0.5	0.5	0.334	0.244	0.176	0.122	0.075
30	0.5	0.5	0.5	0.5	0.467	0.298	0.206	0.136	0.079	0.030
20	0.5	0.5	0.5	0.5	0.400	0.240	0.15	0.072	0.011	0
10	0.5	0.5	0.5	0.5	0.266	0.123	0.019	0	0	0

Table 5
Location of firm 1, x_1^E, depending on the values of F_2 and c when $r = 0.1, \alpha = 0.098, \bar{u} = 4$.

F_2/c	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3
200	0.5	0.5	0.420	0.305	0.235	0.184	0.144	0.112	0.084	0.061
190	0.5	0.5	0.417	0.302	0.231	0.180	0.140	0.108	0.080	0.057
180	0.5	0.5	0.413	0.298	0.228	0.177	0.136	0.104	0.076	0.053
170	0.5	0.5	0.409	0.294	0.224	0.173	0.132	0.099	0.072	0.048
160	0.5	0.5	0.404	0.290	0.220	0.168	0.128	0.095	0.067	0.043
150	0.5	0.5	0.400	0.286	0.215	0.164	0.123	0.090	0.062	0.038
140	0.5	0.5	0.395	0.281	0.210	0.159	0.118	0.085	0.056	0.032
130	0.5	0.5	0.389	0.276	0.205	0.153	0.112	0.079	0.050	0.026
120	0.5	0.5	0.383	0.271	0.200	0.147	0.106	0.072	0.044	0.019
110	0.5	0.5	0.377	0.265	0.193	0.141	0.099	0.065	0.037	0.012
100	0.5	0.5	0.370	0.258	0.186	0.133	0.092	0.058	0.029	0.004
90	0.5	0.5	0.362	0.250	0.178	0.125	0.083	0.049	0.019	0
80	0.5	0.5	0.353	0.242	0.170	0.116	0.074	0.039	0.009	0
70	0.5	0.5	0.341	0.232	0.159	0.105	0.062	0.027	0	0
60	0.5	0.5	0.331	0.220	0.147	0.092	0.049	0.013	0	0
50	0.5	0.5	0.317	0.206	0.132	0.077	0.033	0	0	0
40	0.5	0.5	0.299	0.188	0.113	0.057	0.012	0	0	0
30	0.5	0.5	0.275	0.163	0.087	0.030	0	0	0	0
20	0.5	0.5	0.239	0.127	0.048	0	0	0	0	0
10	0.5	0.399	0.173	0.056	0	0	0	0	0	0

Table 6
Locations of firms 1 and 2, when $r = 0.1,\ \alpha,\ \bar{u},\ F_2$ or c increases.

	$r\nearrow$	$\alpha\nearrow$	$\bar{u}\nearrow$	$F_2\nearrow$	$c\nearrow$
x_1^E	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$
x_2^E	1	1	1	1	1

Note: $x_1^E \nearrow$ means that the location of firm 1 approaches 0.5 (the center), whereas $x_1^E \searrow$ the location of firm 1 approaches 0 (the edge).

Table 7
Values of the three terms in Eq. (21) when $r = 0.1, \bar{u} = 4,\ F_2 = 50$, and $c = 1$.

	x_1^E	First term	Second term	Third term	Sum
$\alpha = 0.0985$	0	40.844	45.943	−107.283	−20.496
$\alpha = 0.0973$	0.192	53.826	42.398	−96.224	0
$\alpha = 0.0969$	0.301	63.135	39.591	−102.694	0
$\alpha = 0.0965$	0.453	80.420	34.584	−115.004	0
$\alpha = 0.092$	0.5	77.479	31.673	−45.367	63.785

Examining the impact of the change in c on the values of the leader and follower requires greater care. We set the parameters as $r = 0.1$ and $\bar{u} = 4$. Fig. 2 demonstrates that the value of the follower is monotonically increasing in c whereas that of the leader does not move monotonically. In a Hoteling setting, transport cost c can be interpreted as the degree of product differentiation. A larger c indicates a greater degree of product differentiation.

In our model, if c increases, the follower always enjoys a positive effect on its total profit, which is not always true for the leader because the effects of an increase in c are classified into three categories: (i) a decrease in T_2, (ii) a decrease in π_1^M and (iii) an increase in π_1^D. In other words, as product space becomes more differentiated, (i) the follower enters earlier, (ii) the monopoly profit decreases and (iii) the duopoly profits after entry increase. The follower only faces (i) and (iii), which are both positive effects for the follower, yielding that V_2 is increasing in c. However, effects (i) and (ii) are negative for the leader, whereas effect (iii) is positive. As c increases, (ii) and (iii), presented by π_1^M and π_1^D respectively, are linearly changed whereas (i), presented by $\tilde{T}_2$, decreases proportionally to $-\log c$. Thus, when c is close to zero, the effect (i) dominates (iii), and V_1 is decreasing in c. When c is relatively large, effect (i) is small, effect (iii) dominates (i) and (ii) and V_1 is increasing in c.

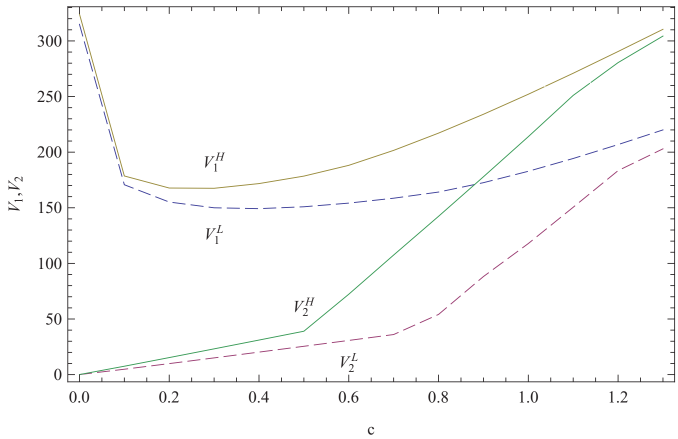

Fig. 2. Plotted values of the values of the leader and the follower with respect to $c \in \{0.001, 0.1, 0.2, 0.3, \ldots, 1.3\}$, when $r = 0.1$, $\bar{u} = 4$. V_1^H and V_2^H are the values of V_1 and V_2 when $\alpha = 0.098$ and $F_1 = F_2 = 60$. V_1^L and V_2^L are the values of V_1 and V_2 when $\alpha = 0.097$ and $F_1 = F_2 = 20$..

In the usual static Hoteling game, only effect (iii) exists, yielding that greater differentiation on a product space always enlarges the values of duopoly firms. This result is well known in the related literature. However, in our dynamic Hoteling game, greater differentiation in a product space may not benefit both firms, thus indicating the importance of introducing timing into the consideration.

5. Discussion and concluding remarks

5.1. Summary and discussion

In this paper, we develop a duopoly model that determines the follower's entry timing, firms' locations and their prices. Examining the timing of investments is important when considering firms' entry strategies. Hence, we extend Lambertini (2002), which in turn extends the location-price competition model (d'Aspremont et al., 1979) by using a continuous time model in which firms earn profits in each instance and the follower's entry timing is given exogenously. Our model endogenizes the follower's entry timing. In doing so, we introduce parameters such as investment costs and a market growth rate to make the model and the outcome more realistic.

We find that these changes create a strategic interaction between the leader's location and the follower's entry timing. As a result, the leader has greater incentive to locate closer to the center to delay the follower's entry. We also find that the follower always locates as far away as possible from the leader, which is a robust result also seen in Lambertini (2002). Numerical analyses are also presented that investigate how the exogenous parameters affect the leader's location and the firms' values. In particular, the profit of the leader changes *non-monotonically* with an increase in the transport cost parameter.

Our paper might be complementary to the finding in Thomadsen (2007) who computationally investigates the location strategies of McDonald's and Burger King in the United States and shows that the quality and cost advantages induce the advantageous firm (McDonald's) to locate around the center and the disadvantageous firm (Burger King) to locate between the center and the market edge in a linear city. These results seem consistent with those in our paper. This means that our paper shows another route to derive the above asymmetric locations in the sense that the first-mover advantage in a dynamic setting is also an important factor of positioning strategies.

Another empirical finding by Thomadsen (2007) is that the distance between the two firms increases as the market area is widened. As shown in Fig. 1 and Tables 4 and 5, our result indicates that the distance between two firms increases as transportation cost c increases. In our model, we normalize the market to length 1, which implies the magnitude of c corresponds to market area, because it signifies the difficulty for consumers to purchase away from their locations. Thus, these results are consistent with each other.

We believe that our setting can be applied to the situations where consumers repeatedly purchase the product, the market is growing and the products are horizontally differentiated. The fast food industry is one of these situations. Leibsohn (2007, p. 46) graphically summarizes the change in the number of McDonald's outlets in the United States. Fig. 3 in his paper shows that the number of its outlets gradually increases. This steady growth of entry implies that the size of the fast food market has gradually grown and that our model prediction would be applicable to this market where products are geographically and/or characteristically differentiated.[21] Other industries that are applicable to our model include telecommunications and pharmaceuticals.[22]

5.2. Future studies

There are four potential future studies. The first addresses the result for which the relationship between the firm values and the transport cost parameter c is nontrivial. c can be interpreted as a parameter that describes the degree of product differentiation. Therefore, c is important for firms and their marketing strategies, necessitating its careful estimation.

Second, we can also consider a second mover advantage in a sense that the follower enjoys benefit from delaying entry. One way this occurs is through the change in the product quality of the follower, allowing the follower to produce a higher quality product as emphasized in Ethiraj and Zhu (2008).[23] Because the quality difference between firms directly influences their location strategies even in a Hoteling model with one period profit opportunity (e.g., Matsumura & Matsushima, 2009; Ziss, 1993), the increase in quality due to entry delay must affect the outcomes in our paper. Another direction of the second mover advantage is technological improvement, letting the follower to adopt the current level of technology with a lower entry cost. This point is considered in the fields of real option and economics as a problem of technology adoption when constructing a continuous time model.[24]

Third, one may consider extending our model by incorporating more than two firms. As shown in previous researches (see, for instance, Brenner, 2005, Fig. 2 on p. 860), when three firms exist, all the firms locate within the linear city in the static setting. This means that the leader's location strategy depends on the number of subsequent firms. We would guess that the leader locates at the center in the three firms case because one of the firms locates at the center in equilibrium in this static case.

The fourth issue, which is theoretically important to consider, is making the entry timing fully endogenous. In this paper, we allow

[21] Toivanen and Waterson (2005) also empirically investigate the entry strategies of McDonald's and Burger King in the UK market where the number of those outlets gradually increased in the period of 1991–1995 (Table 2 in their paper).

[22] These industries have products that are often treated as horizontally differentiated in related researches (see, e.g., Brekke, Königbauer, & Straume, 2007; Foros & Hansen, 2001). Pharmaceutical companies are able to horizontally differentiate their products along two dimensions: indications and side-effects, that is, efficacy and safety (Ethiraj & Zhu, 2008). Products of internet service providers can be differentiated among them through including some products, e.g. Cable-TV by cable-TV-access suppliers, a subsidy of telephone service by telephone providers, and so on (Foros & Hansen, 2001). The growth of market size is discussed in Footnote 14.

[23] They empirically investigate how entry timing of followers (imitators) and horizontal/vertical product differentiation among products influence the relative profit levels of the first mover (the innovator) and the followers, and show a positive relationship between delaying entry (imitation time lag) and the likelihood of which the followers gain higher profits than the first mover. They also show that the first mover keeps its higher profitability under higher horizontal differentiation.

[24] For a very simple extension of our model with technological improvement, see the Appendix.

the follower to enter endogenously; however, the game starts when the leader enters at $T = 0$, thus exogenously. This setting is intentional in order to compare our results to Lambertini (2002). However, we may allow the leader to endogenously enter and, more interestingly, allow a preemption by the firms to enable full analysis of the endogenous entry timing model, as indicated by Fudenberg and Tirole (1985). These are the extensions that are worth considering for future research.

Appendix

Proof of Lemma 1:

The duopoly result is a straightforward maximization of profit by the two firms, as previously stated. See, for example, d'Aspremont et al. (1979).

As for a monopoly, firm 1 is located at or to the left of 1/2, which we denote as x_1. Firm 1 sets up its price so that the consumer at 1 or at a point to the left of 1 is indifferent between purchasing and not purchasing. That is, the consumer at 1 has no surplus, as otherwise firm 1 has an incentive to increase its price without losing any of its consumers. Let this indifferent consumer be denoted $\hat{x}$.

Firm 1's profit is given by

$$\pi_1 = p_1\hat{x},$$
$$s.t. \quad \bar{u} - p_1 - c(\hat{x} - x_1)^2 = 0.$$

Substituting p_1 of the constraint into the profit and maximizing with respect to $\hat{x}$ gives

$$\frac{\partial \pi_1\big|_{p_1}}{\partial \hat{x}} = \bar{u} - c(3\hat{x} - x_1)(\hat{x} - x_1) > 0,$$

from Assumption 1. Thus, the firm sets $\hat{x}$ to be at 1 and we have the monopoly result.

Finally, we must show that the monopoly and duopoly results are sensitive to time only with respect to the entry timing. We show in three steps that this point holds.

First, after the follower enters the market (at a given T_2), the corresponding profits are independent of time t. That is, firm 1 maximizes the following with respect to p_{1t}:

$$\max_{p_{1t}} \int_0^{\bar{x}} p_{1t}(x; x_1, x_2, p_{2t})dx$$

and firm 2 maximizes the following with respect to p_{2t}:

$$\max_{p_{2t}} \int_{\bar{x}}^1 p_{2t}(x; x_1, x_2, p_{1t})dx.$$

The result is the aforementioned simple duopoly.

Second, given this result, firm 2 determines T_2. Firm 2 maximizes Eq. (3) with respect to T_2, and notably, this process does not depend on how firm 1 sets its prices before firm 2 enters the market.

Third, because firm 1's price before firm 2's entry, $p_{1t}, t \in [0, T_2)$, does not affect post-entry competition (the first step) or entry timing (the second step), firm 1 maximizes the following with respect to p_{1t}:

$$\max_{p_{1t}} \int_0^1 p_{1t}(x; x_1)dx.$$

Therefore, the profit maximization problem corresponds to the maximization of the integrand. Therefore, firm 1 achieves the monopoly instantaneous profit flow as previously given.

Hence, we show that the maximization of the instantaneous profit flows is equivalent to the maximization of total profits.

Proof of Proposition 2.

We examine the right hand side of Eq. (21) hereafter. Note that $d\tilde{T}_2(x_1)/dx_1 = (5 - 3x_1)/[\alpha(3 - x_1)(1 - x_1)] > 0$, $\pi_1^M(x_1) - \pi_1^D(x_1, 1) > 0$, $d\pi_1^M(x_1) = 2c(1 - x_1) > 0$ and $d\pi_1^D(x_1, 1) = -c(4 - 3x_1 + x_2)(4 - x_1 - x_2)/18 < 0$.

(a) As α approaches r, $e^{-(r-\alpha)\tilde{T}_2(x_1)}$ approaches 1. The first term is finite. The second term has both the numerator and the denominator approach zero. Thus we use l'Hôpital's rule.

$$\lim_{a\to r} \frac{1 - \exp\left(-(r-\alpha)\tilde{T}_2(x_1)\right)}{r-\alpha} = \lim_{a\to r} \frac{1 - J^{[-(r-\alpha)/\alpha]}}{r-\alpha}$$
$$= \lim_{a\to r} \frac{-rJ^{[-(r-\alpha)/\alpha]}\log J/\alpha^2}{-1} = \log J/r,$$

where $J = \dfrac{18rF_2}{c(1-x_1)(3-x_1)}$,

and thus the second term is also finite. The third term has the numerator equal to 1 and the denominator approaching 0. Thus, this term determines the sign of the whole equation and is negative. Hence, $\partial V_1/\partial x_1$ is negative, making the optimal location for firm 1 to be 0.

(b) As α approaches 0, $\tilde{T}_2(x_1)$ approaches infinity. Therefore, $e^{-(r-\alpha)\tilde{T}_2(x_1)}$ approaches 0. (Note that the derivative of $\tilde{T}_2$ in the first term is unaffected here.) The first and last terms approach 0. The second term does not, and is positive. Hence $\partial V_1/\partial x_1$ is positive, making the optimal location for firm 1 to be 1/2.

(c) Since this equation is continuous with respect to α, (a) and (b) imply that there is some $\hat{\alpha}$ taking a value between 0 and r such that $\partial V_1/\partial x_1$ is equal to 0. In this case, it is optimal for firm 1 to locate strictly inside 0 and 1/2.

Discussion on technological improvement:

In order to investigate the effect of a technological improvement on Proposition 1 (the timing of the follower), we examine the following modified model:

If the fixed cost of the follower decreases as market time increases, the value of the follower V_2 can be rewritten as follows:

$$V_2(T_2, x_1, x_2, p_{1t}, p_{2t}) = \int_{T_2}^{\infty} \pi_{2t}^D(x_1, x_2)e^{-(r-\alpha)t}dxdt - F_2(T_2)$$
$$= \int_{T_2}^{\infty} \pi_{2t}^D(x_1, x_2)e^{-(r-\alpha)t}dxdt - (F_2e^{-\beta T_2})e^{-\alpha T_2},$$

where β denotes the decreasing rate of the fixed cost. An example of an interpretation on β is technological improvement, and this example is adopted by researchers in the field of research and development (R&D).

Considering this setting and deriving the optimal timing of the follower, we have:

$$\tilde{T}_2(x_1) = \frac{1}{\alpha+\beta}\log\left[\frac{(r+\beta)F_2}{\pi_2^D(x_1, 1)}\right].$$

If a new assumption $F_2 > \pi_2^D(x_1, 1)/(r+\beta)$ is made instead of Assumption 2(iii), we can obtain results similar to Proposition 1 and Corollaries 1 and 2, which state that $\tilde{T}_2(x_1)$ is increasing in F_2 or r and decreasing in α. Thus, our model can be applied to the situation with technological improvement.

References

Azevedo, A., & Paxson, D. (2014). Developing real option game models. *European Journal of Operational Research, 237*(3), 909–920.
Biscaia, R., & Mota, I. (2013). Models of spatial competition: a critical review. *Papers in Regional Science, 92*(4), 851–871.
Brekke, K. R., Königbauer, I., & Straume, O. R. (2007). Reference pricing of pharmaceuticals. *Journal of Health Economics, 26*(3), 613–642.
Brenner, S. (2005). Hotelling games with three, four, and more players. *Journal of Regional Science, 45*(4), 851–864.
Brown, S. (1989). Retail location theory: the legacy of Harold Hotelling. *Journal of Retailing, 65*(4), 450–470.

Chronopoulos, M., De Reyck, B., & Siddiqui, A. (2014). Duopolistic competition under risk aversion and uncertainty. *European Journal of Operational Research, 236*(2), 643–656.
d'Aspremont, C., Gabszewicz, J. J., & Thisse, J.-F. (1979). On Hotelling's 'stability in competition'. *Econometrica, 47*(5), 1145–1150.
Dixit, A. K., & Pindyck, R. S. (1994). Investment under uncertainty. Princeton: Princeton University Press.
Ethiraj, S. K., & Zhu, D. H. (2008). Performance effects of imitative entry. *Strategic Management Journal, 29*(8), 797–817.
Foros, Ø., & Hansen, B. (2001). Competition and compatibility among internet service providers. *Information Economics and Policy, 13*(4), 411–425.
Fudenberg, D., & Tirole, J. (1985). Preemption and rent equalization in the adoption of new technology. *Review of Economic Studies, 52*(3), 383–401.
Götz, G. (2005). Endogenous sequential entry in a spatial model revisited. *International Journal of Industrial Organization, 23*(3-4), 249–261.
Hotelling, H. (1929). Stability in competition. *Economic Journal, 39*(153), 41–57.
Kalyanaram, G., Robinson, W. T., & Urban, G. L. (1995). Order of market entry: established empirical generalizations, emerging empirical generalizations, and future research. *Marketing Science, 14*(3s), G212–G221.
Kress, D., & Pesch, E. (2012). Sequential competitive location on networks. *European Journal of Operational Research, 217*(3), 483–499.
Krishnan, V., & Ulrich, K. T. (2001). Product development decisions: a review of the literature. *Management Science, 47*(1), 1–21.
Lages, F. L., & Fernandes, J. C. (2005). The SERPVAL scale: a multi-item instrument for measuring service personal values. *Journal of Business Research, 58*(11), 1562–1572.
Lambertini, L. (1997). Unicity of the equilibrium in the unconstrained Hotelling model. *Regional Science and Urban Economics, 27*(6), 785–798.
Lambertini, L. (2002). Equilibrium locations in a spatial model with sequential entry in real time. *Regional Science and Urban Economics, 32*(1), 47–58.
Lancaster, K. (1990). The economics of product variety: a survey. *Marketing Science, 9*(3), 189–206.
Leibsohn, D. L. (2007). Foreign market development as well as entry: an empirical analysis of two fast-food chains (Ph.D. thesis). University of Michigan.
Malthus, T. R. (1798). An essay on the principle of population. London: Oxford World's Classics.
Matsumura, T., & Matsushima, N. (2009). Cost differentials and mixed strategy equilibria in a Hotelling model. *Annals of Regional Science, 43*(1), 215–234.
Meza, S., & Tombak, M. (2009). Endogenous location leadership. *International Journal of Industrial Organization, 27*(6), 687–707.
Neven, D. J. (1987). Endogenous sequential entry in a spatial model. *International Journal of Industrial Organization, 5*(4), 419–434.
Prescott, E. C., & Visscher, M. (1977). Sequential location among firms with foresight. *The Bell Journal of Economics, 8*(2), 378–393.
Thomadsen, R. (2007). Product positioning and competition: the role of location in the fast food industry. *Marketing Science, 26*(6), 792–804.
Toivanen, O., & Waterson, M. (2005). Market structure and entry: where's the beef? *RAND Journal of Economics, 36*(3), 680–699.
Vakratsas, D., & Kolsarici, C. (2008). A dual-market diffusion model for a new prescription pharmaceutical. *International Journal of Research in Marketing, 25*(4), 282–293.
Vakratsas, D., Rao, R. C., & Kalyanaram, G. (2003). An empirical analysis of follower entry timing decisions. *Marketing Letters, 14*(3), 203–216.
Victor, N. M., & Ausubel, J. H. (2002). DRAMs as model organisms for study of technological evolution. *Technological Forecasting and Social Change, 69*(3), 243–262.
Ziss, S. (1993). Entry deterrence, cost advantage and horizontal product differentiation. *Regional Science and Urban Economics, 23*(4), 523–543.