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NOTES, COMMENTS, AND LETTERS TO THE EDITOR

A Note on Price and Quantity Competition in Differentiated Oligopolies

Jonas Häckner

Department of Economics, Stockholm University, S-106 91 Stockholm, Sweden
jonas.hackner@ne.su.se

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In this note we show that the results developed in N. Singh and X. Vives (1984, *Rand J. Econ.* **15**, 546–554) are sensitive to the duopoly assumption. If there are more than two firms, prices may be higher under price competition than under quantity competition. This will be the case if quality differences are large and goods are complements. If goods are substitutes, high-quality firms may earn higher profits under price competition than under quantity competition. Hence, it is not evident which kind of competition is more efficient. *Journal of Economic Literature* Classification Numbers: D43, L13.

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1. INTRODUCTION

The work of Singh and Vives [3] is a classic contribution to oligopoly theory. It discusses the nature of competition in Bertrand and Cournot markets using the duopoly framework developed by Dixit [1]. The conclusions are strong and clear-cut. Cournot competition always yields higher prices and lower welfare compared to Bertrand competition. When goods are substitutes, firm profits are higher under Cournot competition while if goods are complements Bertrand competition is more profitable. Finally, it is a dominant strategy for firms to choose quantity as their strategic variable when goods are substitutes, and prices when they are complements.

Considering the theoretical and practical importance of these results it seems important to test their robustness with respect to alternative market structures. We therefore extend the Dixit [1] model to allow for an arbitrary number of firms. The discussion is limited to prices and profit levels. The welfare issue becomes too complex in a general setting and the choice of strategic variable is probably better understood as a result of technological factors (Kreps and Scheinkman [2]).

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For simplicity, we allow only for two dimensions of firm heterogeneity, vertical product differentiation and substitutability.[1] Moreover, it is assumed that the marginal cost of production is equal across firms.

The Dixit [1] model generalizes nicely despite the asymmetry that stems from vertical product differentiation. Firm demand depends only on the *average* quality of the competitors' products but is unaffected by changes in the exact distribution of qualities across firms.

The duopoly results do not to generalize to the n-firm setting if quality differences are large. When goods are complements (and quality differences large) low-quality firms will charge prices that are higher under Bertrand competition than under Cournot competition. If goods are substitutes, high-quality firms may earn higher profits under Bertrand competition than under Cournot competition.

2. THE MODEL

The utility function in Singh and Vives [3] is of the type

$$U(q_1, q_2, I) = \alpha_1 q_1 + \alpha_2 q_2 - \tfrac{1}{2}(\beta_1 q_1^2 + \beta_2 q_2^2 + 2\gamma q_1 q_2) + I.$$

For simplicity let us assume that $\beta_1 = \beta_2 = 1$. Thus, utility is quadratic in the consumption of q-goods and linear in the consumption of other goods, I. The parameter $\gamma \in [-1, 1]$ measures the substitutability between the products. If $\gamma = 0$, each firm has monopolistic market power, while if $\gamma = 1$, the products are perfect substitutes. A negative γ implies that the goods are complementary. Finally, α_i measures quality in a vertical sense. Other things equal, an increase in α_i increases the marginal utility of consuming good i.

It is straightforward to generalize the utility function to allow for n firms producing one product variety each,

$$U(\mathbf{q}, I) = \sum_{i=1}^{n} q_i \alpha_i - \tfrac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i \neq j} q_i q_j\right) + I.$$

Consumers maximize utility subject to the budget constraint $\sum p_i q_i + I \leqslant m$, where m denotes income and the price of the composite good is normalized to one. The first-order condition determining the optimal consumption of good k is

$$\frac{\partial U}{\partial q_k} = \alpha_k - q_k - \gamma \sum_{j \neq k} q_j - p_k = 0. \tag{1}$$

[1] When goods are substitutes, the degree of substitutability could be interpreted in terms of horizontal product differentiation.

2.1. *Cournot Competition.* Firm k's inverse demand function can be solved for directly from Eq. (1),

$$p_k(q_k, \mathbf{q}_{-i}) = \alpha_k - q_k - \gamma \sum_{j \neq k} q_j .$$

Firms set quantities to maximize profits, π_k, taking the other firms' quantities as given. If costs are normalized to zero, firm k's reaction function equals

$$q_k(\mathbf{q}_{-k}) = \frac{\alpha_k - \gamma \sum_{j \neq k} q_j}{2} .$$

Summing over all firms we arrive at

$$\sum_{i=1}^{n} q_i = \frac{\sum_{i=1}^{n} \alpha_i - \gamma(n-1) \sum_{i=1}^{n} q_i}{2}, \tag{2}$$

Finally, noting that

$$\begin{aligned} \sum_{i=1}^{n} q_i &= q_k + \sum_{j \neq k} q_j \\ \sum_{i=1}^{n} \alpha_i &= \alpha_k + \sum_{j \neq k} \alpha_j \end{aligned} \tag{3}$$

we can solve for demand and price in equilibrium

$$q_k^C = p_k^C = \frac{\alpha_k[\gamma(n-2)+2] - \gamma \sum_{j \neq k} \alpha_j}{(2-\gamma)[\gamma(n-1)+2]} .$$

Thus, firm k's equilibrium price and quantity depend on the average quality of its competitors but are independent of the exact distribution of qualities across firms.

2.2. *Bertrand Competition.* Summing over all firms, Eq. (1) can be written

$$\sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n} q_i - \gamma(n-1) \sum_{i=1}^{n} q_i - \sum_{i=1}^{n} p_i = 0. \tag{4}$$

Equations (1), (3), and (4) then yield firm k's demand function,

$$q_k(p_k, \mathbf{p}_{-k}) = \frac{(\alpha_k - p_k)[\gamma(n-2)+1] - \gamma \sum_{j \neq k} (\alpha_j - p_j)}{(1-\gamma)[\gamma(n-1)+1]} . \tag{5}$$

Profit maximization (and zero costs) then implies the reaction function

$$p_k(\mathbf{p}_{-k}) = \frac{\alpha_k}{2} - \frac{\gamma \sum_{j \neq k} (\alpha_j - p_j)}{2[\gamma(n-2)+1]}. \tag{6}$$

Summing this over all firms we arrive at

$$\sum_{i=1}^{n} p_i = \frac{\sum_{i=1}^{n} \alpha_i}{2} - \frac{\gamma(n-1) \sum_{i=1}^{n} (\alpha_i - p_i)}{2[\gamma(n-2)+1]}. \tag{7}$$

Equations (3), (6), and (7) then yield the equilibrium prices and quantities for firm k,

$$p_k^B = \frac{\alpha_k[\gamma^2(n^2-5n+5)+3\gamma(n-2)+2] - \gamma \sum_{j \neq k} \alpha_j[\gamma(n-2)+1]}{[\gamma(n-3)+2][\gamma(2n-3)+2]}$$

$$q_k^B = \frac{[\alpha_k[\gamma^2(n^2-5n+5)+3\gamma(n-2)+2] - \gamma \sum_{j \neq k} \alpha_j[\gamma(n-2)+1]][\gamma(n-2)+1]}{(1-\gamma)[\gamma(n-3)+2][\gamma(n-1)+1][\gamma(2n-3)+2]}.$$

Again, firm k's equilibrium price and quantity depend on the average quality of its competitors but are independent of the exact distribution.

3. BERTRAND AND COURNOT EQUILIBRIA

The first proposition in Singh and Vives [3] deals with the welfare implications of price and quantity competition. Cournot prices are found to be higher than Bertrand prices regardless of whether goods are complements or substitutes. This means that welfare is always higher under price competition. Moreover, Cournot profits are shown to be higher than Bertrand profits when goods are substitutes. If goods are complements, the opposite relation holds.

If we take the difference between the Cournot price and the Bertrand price of the generalized model we arrive at the condition

$$\begin{aligned} &sign(p_k^C - p_k^B) \\ &\quad = sign\left(\alpha_k[\gamma^2(n^2-5n+5)+4\gamma(n-2)+4] - \gamma^2(n-2) \sum_{j \neq k} \alpha_j\right). \end{aligned}$$

Hence, unless $n=2$, the price differential consists of a positive and a negative term. The sign will be determined by the number of firms, the degree of horizonal differentiation and the quality distribution between firm k and its competitors.

PROPOSITION 1. *Assume that* $n > 2$. (i) *When goods are substitutes, prices are higher under Cournot competition than under Bertrand competition. Hence, welfare is unambiguously higher when firms compete in prices.* (ii) *When goods are complements, and quality differences are large, low-quality firms charge higher prices under Bertrand competition than under Cournot competition.*

Proof. First consider the case when goods are substitutes. Let z_k be the ratio between the average quality offered by the rival firms, $\sum \alpha_j/(n-1)$, and the quality offered by firm k, α_k. If $z_k < 1$ firm k produces a better product than the average rival and vice versa. Unless z_k is small enough, firm k will be driven out of the market in equilibrium. Specifically, under Bertrand competition $q_k^B > 0$ if

$$z_k < \frac{\gamma^2(n^2-5n+5)+3\gamma(n-2)+2}{\gamma(n-1)(\gamma(n-2)+1)} \equiv z^B$$

while the corresponding condition under Cournot competition is

$$z_k < \frac{\gamma(n-2)+2}{\gamma(n-1)} \equiv z^C.$$

The second-order condition for an interior solution under Bertrand competition is $c > 1/(1-n)$. This ensures that $z^B < z^C$. Hence, it is not meaningful to talk about an n-firm market unless $z_k < z^B$.

The price differential is positive unless z_k is large. Specifically, $p_k^C - p_k^B > 0$ unless

$$z_k > \frac{\gamma^2(n^2-5n+5)+4\gamma(n-2)+4}{\gamma^2(n-2)(n-1)} \equiv z_k^*.$$

It is straightforward to verify that there exists no z_k that can satisfy the latter and the former inequalities simultaneously if $\gamma > 0$. This proves the first part of the proposition.

Assume now that goods are complements and that the second-order conditions are satisfied, i.e., $\gamma \in [1/(1-n), 0]$. Then, all firms face a positive demand in equilibrium regardless of the distribution of qualities. Consequently, $z_k > z_k^*$ is a sufficient condition for a negative price differential. It can be checked that z_k^* increases monotonically in γ. Moreover, $z_k^* > 0$ when $\gamma = 1/(1-n)$ and z_k^* approaches $+\infty$ as γ approaches zero from below. Hence, for every γ in the interval $[1/(1-n), 0]$ there exists a threshold value z_k^* such that $z_k > z_k^*$ implies that $p_k^C < p_k^B$. Finally, z_k^* decreases in n and approaches $z_k^* = 1$ as n approaches infinity. Hence, only firms producing qualities below average will charge higher prices under Bertrand competition than under Cournot ompetition. ▌

A switch from Cournot to Bertrand competition will reduce the prices of high-quality products and increase equilibrium demand. If goods are complements, this will increase the demand for low-quality products which, in turn, may enable low-quality producers to raise their prices.

PROPOSITION 2. (i) *When goods are complements, Bertrand profits are higher than Cournot profits.* (ii) *When goods are substitutes, and quality differences are small, Cournot profits are higher than Bertrand profits. If quality differences are large, high-quality firms may earn higher profits under Bertrand competition.*

Proof. First define $G(z_k) = \pi_k^C/\pi_k^B$ and let z^C and z^B be defined as in the proof of Proposition 1. If $\gamma = 0$ then $G(z_k) = 1$. When goods are substitutes, $G(z_k)$ increases in z_k. $G(1) > 1$ and $G(z^B) = +\infty$. Hence, only high-quality firms (i.e., firms with a low z_k) may have a profit ratio that is smaller than one. For instance, suppose that $\alpha_j = z_k \alpha_k$, $\forall j \neq k$. Then, let $\hat{z}$ denote the lowest value of z_k for which there is a positive demand for low-quality products in the Bertrand equilibrium. $G(\hat{z}) < 1$ as long as

$$\gamma < \hat{\gamma}(n) \equiv \frac{\sqrt{n^4 - 2n^3 - 5n^2 + 14n - 7} + n^2 - 5n + 5}{2(n^2 - 3n + 2)}.$$

$\hat{\gamma}$ is increasing in n. $\hat{\gamma}(2) = 0$ while $\hat{\gamma}(\infty) = 1$. Hence, for any market structure there exist a γ small enough to make $G(\hat{z}) < 1$. This, in turn, implies that there exists some positive interval $[\hat{z}, \bar{z}]$ such that $G(z_k) < 0$ if $z_k \in [\hat{z}, \bar{z}]$.

When goods are complements $G(z_k)$ decreases in z_k. Since $G(0) < 1$ the profit ratio is always smaller than one. ∎

When quality differences are large, high-quality firms become insulated from competition from the low-quality segment. Therefore, price competition may not hurt firm profits more than quantity competition.

4. CONCLUDING REMARKS

We may conclude that the results in Singh and Vives [3] are sensitive to the duopoly assumption. Although we have imposed more symmetry on the model, compared to the original formulation, the clear-cut dichotomy between Bertrand and Cournot competition is lost in the n-firm specification. Hence, it is not evident which type of competition is more efficient.

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