Regional Science and Urban Economics 29 (1999) 463–472

Cooperation vs. competition in a spatial model

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Received 15 January 1997; received in revised form 12 January 1999; accepted 27 January 1999

Abstract

This paper discusses the element of cooperation between firms in the form of information exchange through communication into the Hotelling spatial competition model. It is shown that subgame perfect equilibrium in a two-stage game can be achieved in a wide range from minimum differentiation to maximum differentiation, depending upon the relative strength of the cooperation effect over the competition effect. This result is exemplified in the economics of the Silicon Valley.

1. Introduction

Hotelling (1929) first proposed what has subsequently been termed the principle of ''Minimum Differentiation'': two firms of a homogeneous product agglomerate at the center of the line market under linear transportation costs. This principle was challenged by D'Aspremont et al. (1979). They demonstrated that when both firms locate together, price competition *à la* Bertrand drives down to zero equilibrium prices and that under quadratic transportation costs the two firms are located at the endpoints of the interval. Since then, a considerable effort has been devoted to restoring the validity of this principle: for example, by introducing enough heterogeneity in both consumers and firms operating as a hidden dimension (De Palma et al., 1985 and Rhee et al., 1992), by fixing the market price exogenously

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0166-0462/99/$ – see front matter
PII: S0166-0462(99)00007-1

and imposing some harmonious conjectural variations (Stahl, 1982), by considering explicitly price collusion (Jehiel, 1992 and Friedman and Thisse, 1993), and by adopting price-matching policy (Zhang, 1995). Because price competition in the spatial competition model is a strong centrifugal force, these efforts have attempted to influence the subgame of price decisions by relaxing price competition. However, no attempt is made to deal with the subgame of location decisions directly.

Over the past decade, a considerable body of research has documented the historical emergence and organizational structure of Northern California's Silicon Valley (including Saxenian, 1984, 1992, 1996; Rogers and Larsen, 1984; and Angel, 1991). In particular, Saxenian (1996, p. 33) emphasized the importance of communication among firms, permitting the exchange of information in the making of the Silicon Valley as an efficient industrial system:[1]

> 'By all accounts, these informal conversations were pervasive and served as an important source of up-to-date information about competitors, customers, markets, and technologies. Entrepreneurs came to see social relationships and even gossip as a crucial aspect of their business. In an industry characterized by rapid technological change and intense competition, such informal communication was often of more value than more conventional but less timely forums such as industrial journals.'

Indeed, the exchange of information within a set of firms generates externality-like benefits to each of them. Since communications typically involve distance-sensitive costs, the benefits are greater if firms locate close to each other. Hence, all other things being equal, firms tend to cluster to ease the flow of communications. In other words, externality is a centripetal force.

The purpose of the present paper is to develop a model of spatial competition *à la* Hotelling (1929) which incorporates a centripetal force (the externality) and a centrifugal force (price competition). In doing so, we shall introduce the element of cooperation in the form of information exchange through communications into the Hotelling model and examine the conditions under which the validity of the principle is restored. In the process, we will also demonstrate how the coexistence of cooperation and competition makes a regional economy like the Silicon Valley a stable industrial system.

The remainder of the paper is organized as follows. In Section 2, we develop a simple duopoly model of spatial competition to investigate the equilibrium locations of two-stage games in which cooperation between firms is formulated. In Section 3, socially optimal locations are analyzed. Section 4 explores the economics of the Silicon Valley. The final section concludes the paper.

[0]For a more detailed discussion, see Fujita and Thisse (1996).

2. The model

The basic model is a variant of Hotelling's (1929) spatial duopoly model. Consumers are uniformly distributed along the unit interval [0,1] and the location of each is denoted by $x \in [0,1]$. The transportation cost incurred by consumers is assumed to be a quadratic function of distance. The full price for a consumer located at x who buys from firm i is $P_i + t(x_i - x)^2$, where P_i is the price of firm i, t is transport rate, and x_i is location of firm i. Each consumer buys one unit of the good from the firm with the lower full price if that full price does not exceed the reservation price. Two firms are producing a homogeneous good at an equal marginal cost $c(F_0)$, which is a function of sunk fixed cost, F_0, with $c'(F_0) < 0$. We can think of F_0 as an irreversible investment in cost-reducing R&D. It is assumed that cooperation between firms takes a particular form: information about R&D is exchanged through communication with one other. An important characteristic of information is its public-good nature: the use of a piece of information does not reduce the usefulness of the information to other firms. It is further assumed that firms symmetrically communicate and they equally split communication cost. The relative spatial proximity of firms, as measured by $-(x_1 - x_2)^2$, makes information about R&D easier and hence reduces R&D costs. In this sense, R&D costs decrease with the distance between firms. In the following, we define $\tau(x_1 - x_2)^2$ as the total communication cost of engaging in the exchange of information about R&D, where t is the externality parameter (or communication cost per unit). Indeed, different industries may involve different degrees of externality. The larger the value of τ, the higher is the technology of the industry[2] and the higher is the externality between two firms in the industry. Thus, cooperation is more likely between firms.

Without loss of generality, we assume $x_1 \leq x_2$ throughout this paper. When firms are set up at $x_1 < x_2$, the marginal consumer, who is indifferent between purchasing from either firm, is located at $\hat{x}$ as given by:

$$\hat{x} = \frac{(P_2 - P_1)}{2t(x_2 - x_1)} + \frac{(x_2 + x_1)}{2} \tag{1}$$

Using Eq. (1), we can derive aggregate demand for firms 1 and 2, respectively, as follows:

$$Q_1 = \int_0^{\hat{x}} \mathrm{d}x = \hat{x} = \frac{(P_2 - P_1)}{2t(x_2 - x_1)} + \frac{(x_2 + x_1)}{2} \tag{2.1}$$

[2]When τ increases, communication costs increase as well, leading to higher opportunity costs for R&D. This, in turn, implies that technology costs in the industry are higher.

$$Q_2 = \int_{\hat{x}}^{1} \mathrm{d}x = 1 - \hat{x} = 1 - \frac{(P_2 - P_1)}{2t(x_2 - x_1)} - \frac{(x_2 + x_1)}{2} \tag{2.2}$$

The firms' profit functions are given, respectively, by:

$$\begin{aligned}\pi_1 &= [P_1 - c(F_0)]\,Q_1 - [F_0 + \tau(x_2 - x_1)^2] \\ &= [P_1 - c(F_0)]\left[\frac{(P_2 - P_1)}{2t(x_2 - x_1)} + \frac{(x_2 + x_1)}{2}\right] - [F_0 + \tau(x_2 - x_1)^2]\end{aligned} \tag{3.1}$$

$$\begin{aligned}\pi_2 &= [P_2 - c(F_0)]\,Q_2 - [F_0 + \tau(x_2 - x_1)^2] \\ &= [P_2 - c(F_0)]\left[1 - \frac{(P_2 - P_1)}{2t(x_2 - x_1)} - \frac{(x_2 + x_1)}{2}\right] - [F_0 + \tau(x_2 - x_1)^2]\end{aligned} \tag{3.2}$$

In the spirit of Hotelling, we study a subgame perfect equilibrium in a two-stage game. In the first stage, both firms simultaneously choose their production locations, x_1 and x_2, respectively. In the second stage, the production locations are known and the two firms choose simultaneously prices P_1 and P_2, respectively. As usual, we proceed by backwards induction and start with the second-stage subgame.

Taking the first derivatives of Eqs. (3.1) and (3.2) with respect to P_1 and P_2, setting them equal to zero, respectively, and then solving the resulting equations simultaneously, we obtain:[3]

$$P_1 = \frac{2t}{3}(x_2 - x_1) + \frac{t}{3}(x_2 - x_1)(x_2 + x_1) + c(F_0) \tag{4.1}$$

$$P_2 = \frac{4t}{3}(x_2 - x_1) - \frac{t}{3}(x_2 - x_1)(x_2 + x_1) + c(F_0) \tag{4-2}$$

Upon substitution of Eq. (4) into Eq. (3), we have:

$$\pi_1^* = \frac{t}{18}(x_2 - x_1)\,(2 + x_1 + x_2)^2 - [F_0 + \tau(x_2 - x_1)^2] \tag{5.1}$$

$$\pi_2^* = \frac{t}{18}(x_2 - x_1)\,(4 - x_1 - x_2)^2 - [F_0 + \tau(x_2 - x_1)^2] \tag{5.2}$$

[3]It is easy to see that the second-order conditions are met:

$$\frac{\partial^2 \pi_1}{\partial P_1^2} = \frac{-1}{t(x_2 - x_1)} < 0$$

$$\frac{\partial^2 \pi_2}{\partial P_2^2} = \frac{-1}{t(x_2 - x_1)} < 0.$$

We now turn to the first-stage game. Taking the first derivatives of Eqs. (5.1) and (5.2) with respect to x_1 and x_2, setting them equal to zero, respectively and then restricting the resulting solution to a symmetric one (i.e., $x_1 + x_2 = 1$), we get:

$$x_1^* = \frac{12\tau - t}{4t + 24\tau} = \frac{12 - \frac{t}{\tau}}{4\frac{t}{\tau} + 24} \tag{6.1}$$

$$x_2^* = \frac{12\tau + 5t}{4t + 24\tau} = \frac{12 + 5\frac{t}{\tau}}{4\frac{t}{\tau} + 24} \tag{6.2}$$

From Eq. (6), it is apparent that the equilibrium location is a function of the ratio $\frac{t}{\tau}$, i.e., the trade-off between a centripetal force (the externality) and a centrifugal force (price competition) as expressed by two parameters τ and t. The comparative static effect of an increase in τ on the equilibrium locations can be easily evaluated as follows:

$$\frac{\partial x_1^*}{\partial \tau} = \frac{72t}{(4t + 24\tau)^2} > 0 \tag{7.1}$$

$$\frac{\partial x_2^*}{\partial \tau} = -\frac{72t}{(4t + 24\tau)^2} < 0 \tag{7.2}$$

From Eqs. (6) and (7), we establish:

Proposition 1. (*i*) *The larger the externality between two firms, the less will be the location differentiation between the two firms.*

(*ii*) *As the externality approaches* ∞, *the agglomeration or minimum location differentiation* (*i.e.,* $x_1^* = x_2^* \approx \frac{1}{2}$) *of the two firms is a market equilibrium.*

(*iii*) *When the externality is relatively small such that* $\tau \leq 1/12t$, *the equilibrium is the 'bounded' corner solution* (*i.e.,* $x_1^* = 0$ *and* $x_2^* = 1$).[4]

(*iv*) *If there is no externality* (*i.e.,* $\tau = 0$), *the equilibrium is reduced to the unbounded solution* (*i.e.,* $x_1^* = -\frac{1}{4}$, $x_2^* = \frac{5}{4}$).[5]

This result is a generalization of spatial competition theory and highlights the possibility of intermediate locations. In particular, it points out that the location difference not only decreases when τ rises, but also when t falls. Under linear

[4]See D'Aspremont et al. (1979) and Neven (1985) for further details.
[5]See Tabuchi and Thisse (1995) for a detailed discussion.

transportation costs, Hotelling (1929) claimed that competition between two sellers of a homogeneous product leads to their agglomeration at the center of a linear, bounded market (i.e., $x_1^* = x_2^* = \frac{1}{2}$). D'Aspremont et al. (1979) challenged this proposition on technical grounds since, with the Hotelling specifications, equilibrium in pure strategies fails to exist as the two firms locate close to each other. Under quadratic transportation costs, they demonstrated that the subgame perfect equilibrium in their two-stage game is maximum differentiation in which the two firms will locate at opposite ends of the bounded market (i.e., $x_1^* = 0$ and $x_2^* = 1$). Furthermore, when the market is unbounded, Tabuchi and Thisse (1995) showed that the two firms choose to locate outside the market (i.e., $x_1^* = -\frac{1}{4}$, $x_2^* = \frac{5}{4}$). This outcome seems to reflect the fact that price competition under quadratic transportation costs is very fierce, leading the firms to establish themselves far away from each other.

It is worth noting that in our analysis, the validity of the principle of minimum differentiation depends on diverse external sources of information and technology rather than on the suppression of price competition. Specifically, a key to the principle of minimum differentiation is cooperation between firms which moderates price competition. In other words, minimum differentiation arises from the firms' cooperation effect: the price competition effect is relegated to a secondary effect by both firms' cooperative actions to exchange information about R&D through close communications with each other. Thus, cooperation becomes a predominant concern for competing firms in their location decisions.

We now move to the effect of a fall in transport rate on equilibrium locations. It immediately follows from Eq. (6) that

$$\frac{\partial x_1^*}{\partial t} = \frac{-72t}{(4t + 24\tau)^2} < 0 \tag{8.1}$$

$$\frac{\partial x_2^*}{\partial t} = \frac{72t}{(4t + 24\tau)^2} > 0 \tag{8.2}$$

leading to the following proposition:

Proposition 2. *Given the degree of externality τ, the lower transportation rate t, the less will be the location differentiation between firms.*

At first glance, this result may seem surprising. In the absence of externality, we see that equilibrium locations as well as prices are independent of transportation rate t as illustrated in traditional location literature.[6] In other words, a centrifugal force, t, plays no role in the determination of equilibrium locations and prices when externality is absent. Nevertheless, when externality exists, a decrease in transportation rate moderates price competition, thereby leading both firms closer to each other.

3. Socially optimal locations

Hotelling (1929) also demonstrated that minimization of the social cost of transportation requires both firms to occupy symmetrical positions at the quartiles of the market. An interesting question naturally arises: what are the optimal locations in the presence of externality? To pursue this issue, we have to define social welfare. As a matter of fact, maximizing social welfare in our specification is equivalent to minimizing social cost including total transportation plus communication costs, which is specified as follows:

$$T = t\left\{\int_0^{x_1} (x_1 - x)^2\,dx + \int_{x_1}^{(x_1+x_2)/2} (x - x_1)^2\,dx + \int_{(x_1+x_2)/2}^{x_2} (x_2 - x)^2\,dx + \int_{x_2}^{1} (x - x_2)^2\,dx\right\} + 2[F_0 + \tau(x_1 - x_2)^2]$$

$$= \frac{1}{3}t\left[x_1^3 + \frac{1}{4}(x_2 - x_1)^3 + (1 - x_2)^3\right] + 2[F_0 + \tau(x_1 - x_2)^2] \tag{9}$$

Differentiating Eq. (9) with respect to x_1, setting it equal to zero, and assuming that the two firms are located symmetrically ($x_1 + x_2 = 1$), then we obtain the candidate optimal locations as follows:

$$x_1^w = \frac{t + 16\tau}{4t + 32\tau} \tag{10.1}$$

$$x_2^w = \frac{3t + 16\tau}{4t + 32\tau} \tag{10.2}$$

[6] Substituting Eq. (6) into Eq. (4), we obtain:

$$P_1^* = P_2^* = \frac{3t}{2t + 6\tau} + c(F_0)$$

indicating that the equilibrium prices are identical and dependent on t and τ. However, we see that when $\tau = 0$, $P_1^* = P_2^* = \frac{3}{2} + c(F_0)$ which is independent of t.

From Eq. (10), we have

$$x_1^w \geq \frac{1}{4} \tag{11.1}$$

$$x_2^w \leq \frac{3}{4} \tag{11.2}$$

Therefore, we can establish:

Proposition 3. *Suppose there exists externality between firms.*

(*i*) *The optimal locations of the two firms are inside the first and third quartiles.*

(*ii*) *The location differentiation between the two firms becomes smaller (greater) as the externality (transportation rate) increases.*

(*iii*) *When τ approaches ∞, the agglomeration or minimum location differentiation (i.e., $x_1 = x_2 \approx \frac{1}{2}$) of the two firms is social optimal and also the market equilibrium solution.*

(*iv*) *When $\tau = 0$, the optimal locations are $x_1^w = \frac{1}{4}$ and $x_2^w = \frac{3}{4}$.*

Obviously, without externality, the socially optimal locations are the first and third quartiles, which are exactly the same as in Hotelling. It should be noted that this result holds true no matter whether transportation costs are linear or quadratic.

4. Implications and conclusions

This paper has developed a simple model of spatial competition *á la* Hotelling by introducing the element of cooperation between firms in the form of information exchange through communication. It is shown that equilibrium locations and prices are determined by the two forces: cooperation and competition. Specifically, equilibrium can be achieved in a wide range from minimum differentiation to maximum differentiation depending upon the relative strength of the cooperation effect over competitive effect. More important, the validity of the principle of minimum differentiation is restored if cooperation (i.e., a centripetal force) becomes a predominant concern.

The present paper has examined the extent to which industrial activity clusters spatially and to link this geographic concentration to the existence of knowledge or information externalities. One of the most important implications is that location and proximity do matter in exploiting knowledge spillovers. The balance between cooperation and competition is the driving principle behind the 'industrial districts', which Marshall announced a century ago, and is most recently embodied in the contemporary industrial clusters such as Silicon Valley (see Zaratiegui, 1997; and You and Wilkinson, 1994). Specifically, cooperation among firms through face-to-face communications plays an important role in high-tech agglomeration in the Silicon Valley. As Saxenian (1996) emphasized, it is the communication between individuals which facilitates the transmission of knowledge across agents, firms, and even industries, and not just the high endowment of worker's knowledge that is conductive to innovative activity. Of particular importance in providing a source of innovative-generating knowledge are research scientists at universities (such as Stanford University). Indeed, there is considerable evidence supporting the existence of spatially-mediated knowledge spillovers. Jaffe (1989) argued that the proximity of university research to corporate laboratories should raise the potency of spillovers from university laboratories. Acs et al. (1992) provided evidence suggesting that the knowledge created in university laboratories spill over to contribute to the generation of commercial innovations in the private sector. Acs et al. (1994) and Feldman (1994a and 1994b) found persuasive evidence that spillovers from university research contribute substantially to the innovative activity of private corporations. Furthermore, not only have Jaffe et al. (1993) found that patent citations tend to occur more frequently within the state in which they were patented than outside of that state, but Audretsch and Feldman (1996) found that the propensity of innovative activity to cluster geographically tends to be greater in industries where new economic knowledge plays a more important role. It is in this sense that our results do highlight the economics of the Silicon Valley.

Acknowledgements

We would like to thank an anonymous referee for some very useful comments. We are also indebted to Professor J.-F. Thisse and T. Tabuchi for their very insightful suggestions. Paper presented at The Fifth World Congress of the Regional Science Association International (RSAI), May 2–6, 1996, Tokyo, Japan.

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