Economics Letters 23 (1987) 323–328
North-Holland

THE ROLE OF INDIFFERENCE IN SEQUENTIAL MODELS OF SPATIAL COMPETITION
An Example

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Received 20 November 1986
Final version received 5 January 1987

This note shows how indifference expands the set of subgame-perfect equilibria in an illustrative model of spatial competition. The model concerns sequential location for three firms on a unit segment, with consumers buying the good from the nearest firm. Indifference occurs because, when the third firm enters in-between the first two, it gets the same payoff independently of its particular location. The note ends with a discussion of a particular equilibrium outcome, the one where the indifferent player is allowed to use his indifference optimally, in order to influence other players' strategies.

1. Introduction

Several recent papers on spatial competition [Prescott and Visscher (1977), Lane (1980) and Eaton and Wooders (1983)] have modeled their problem using multi-stage games. The analysis concentrates on sequential or subgame-perfect equilibria, which allow to rule out incredible threats, and to use backward programming to find the (hopefully unique) equilibrium of the game.

Incredible threats are threats a player would not find in his interest to carry out, because other moves are *strictly* better for him at that particular stage of the game. The word 'strictly' is crucial since, when there is *indifference* between several moves, there is a breakdown of the usual backward-induction argument: a player's optimal strategy will usually depend on how the indifferent players moving after him will resolve their ties. The set of equilibria is expanded because the indifference can be resolved in different ways, which in turn affects other players' payoffs and strategies.

This point is important in the spatial competition literature, where indifference can naturally arise. Section 2 illustrates this problem, through a simple example in Hotelling's tradition, where the market consists of a continuum of consumers uniformly distributed on a linear segment and of three firms entering sequentially [this model was first analyzed by Prescott and Visscher (1977)]. Consumers earch buy one unit of output from the nearest firm, and the element of indifference is that, when the third firm enters in-between the other two, it gets the same market share independently of the particular location choice. The example is very simple, which allows us to fully characterize the equilibrium set. We suggest moreover how the indifference problem also appears in other models of spatial competition.

The existence of multiple equilibria leads to the general issue of the choice of a particular outcome. One way of resolving the indeterminacy would be to allow players to choose 'optimal' ways to resolve their ties. Indeed, if a player is indifferent between two moves, he can still hope to influence his payoff by the consequences his strategies have on other players, and he will thus care

* We thank Andreu Mas-Colell and Dilip Abreu for comments.

0165-1765/87/$3.50

about how his rivals perceive the ties will be resolved. Section 3 briefly discusses the extent to which this idea can be used to choose a particular equilibrium.

2. Example

2.1. Basic setup

The example concerns a pure location model on a unit segment, with three firms, A, B and C, entering sequentially (in that order). We assume exogenously fixed prices, and consumers (distributed uniformly on the segment) who buy exactly one unit of the good from the nearest firm. The *payoffs* are non-negative numbers (a, b, c) such that $a+b+c=1$. The cost of entering the market and the marginal cost of production are zero, but relocation is prohibitively costly.

We restrict ourselves to subgame-perfect equilibria, which amounts to saying that firms expect their rivals to act in order to maximize profits conditional on previous moves of the game. We concentrate only on pure strategies. Such *strategies* take the following form for each firm:

- Firm A's strategies are locations $x_A \in [0; 1]$.
- Firm B's strategies are functions associating a location x_B to each possible x_A.
- Firm C's strategies are functions associating a location x_C to each possible pair $(x_A; x_B)$.

2.2. Element of indifference

Solving this location problem by backward induction involves computing first the optimal x_C for C given each $(x_A; x_B)$, then the optimal x_B for B given x_A and C's optimal reaction to $(x_A; x_B)$, and finally A's optimal x_A given B and C's optimal reactions. This argument is, however, plagued here by the difficulty that, whenever it is optimal for C to locate in-between A and B, any particular location in this interval is as good as any other (C will get in any case one-half of it as market share). In order to rescue the backward-induction mechanism, Prescott and Visscher (1977) restrict C's strategy set by assuming that, in that case, it will always locate at the middle-point of the interval. They also assume this restriction to be common knowledge, which gives them the unique equilibrium location ($x_A=0.25$, $x_B=0.75$, $x_C=0.5$), with A and B each receiving 0.375, and C getting 0.25. As they stress, this restriction is arbitrary. We now drop it, and compute the set of sustainable outcomes, starting first by ruling out some allocations.

2.3. Narrowing the set of potential equilibria

The following restrictions can be derived from individual rationality:

- *Firm C should not get less than 0.25 in equilibrium*: it will go either in-between A and B, or just 'next to them',[1] depending on $0.50 \cdot (x_B - x_A) \gtrless \max\{x_A; 1-x_B\}$ (assuming without loss of generality $x_A \leq x_B$). In order to choose $x_C \in [x_A; x_B]$, $x_B - x_A$ must equal at least 0.50, which gives C at least 0.25. When $x_B - x_A < 0.50$, then $\max\{x_A; 1-x_B\}$ is also at least 0.25.
- *Firm A should not get less than 0.25 in equilibrium*: by choosing $x_A = 0.25$, $x_B \geq 0.75$ is optimal for firm B, and the worst that can happen to A is having C just next to its right, which gives A 0.25. Assuming, without loss of generality, $x_A \leq 0.50$, it follows that A will always choose $x_A \leq 0.25$: if

[1] We assume a firm can locate arbitrarily close to another, so that, for example, locating 'just to the left' of $x_A (\leq x_B)$ gives firm C the total segment $[0; x_A]$.

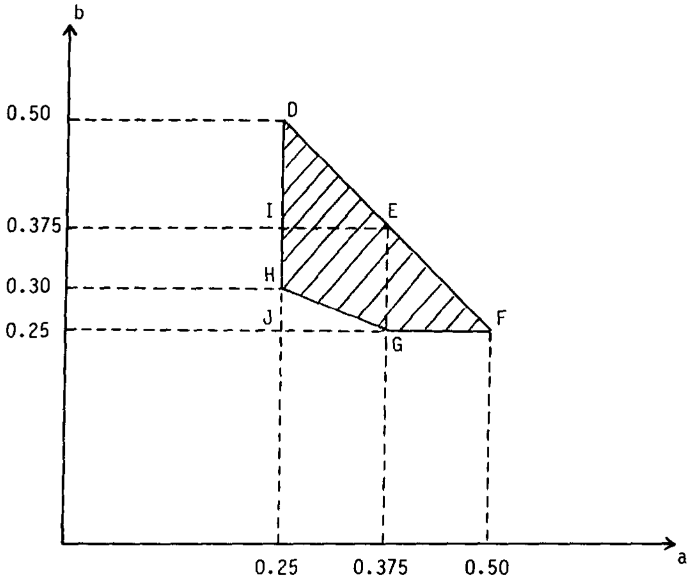

Fig. 1. Set of subgame-perfect equilibrium allocations.

$x_A > 0.25$, B will find it profitable to send C just to the left of A by choosing $x_B = 1 - x_A + \epsilon$, with ϵ arbitrarily small. This would give A strictly less than 0.25.

- *Firm B should not get less than 0.25 in equilibrium*: firm B can guarantee itself at least $(1 - x_A)/3$ by going at $x_B = x_A + 2 \cdot (1 - x_A)/3$ (at worst, C will locate just next to it). With $x_A \leq 0.25$, this yields a market share of at least 0.25 to B.

At this point, we are left with the set of potential equilibrium payoffs represented by area *DJF* (see fig. 1, where C's share equals $1 - a - b$). Not all these outcomes can be sustained, however. For example, we saw that B can get by itself at least $(1 - x_A)/3$. Giving B a share of 0.25 thus implies $x_A = 0.25$. The best C can get is then 0.375 (when $x_B = 1$). This implies $x_C = 0.50$ (in order to give B a share of 0.25), and thus A should receive at least 0.375 when B gets 0.25.

It is when A gets 0.25 that C can get its best market share: the constraints on C's market share c are: $c \leq (1 - x_A)/2$ (maximum when $x_B = 1$) and $c \leq 1 - 0.25 - (1 - x_A)/3$ (where 0.25 is A's market share, and $(1 - x_A)/3$ is the minimum B can guarantee itself). The idea is that a higher x_A lowers the joint market share of B and C, but at the same time lowers B's minimum market share. From C's point of view, one can verify that the optimum is $x_A = 0.10$, $x_B = 1.00$ and $x_C = 0.40$ (this gives A a market share of 0.25, B a share of 0.30, which is $(1 - x_A)/3$, and C a share of 0.45).

We are thus left with a set of potential equilibrium payoffs represented by the shaded area *DHGF* of fig. 1.

2.4. Sustainability

We now show that the entire shaded area of fig. 1 represents perfect equilibria of the location game, depending on C's strategic use of its indifference:

Segment D–F is the one where firm C is worst off (it gets only 0.25). This can be sustained if C chooses $x_C = x_A + \alpha \cdot (x_B - x_A)$, with $\alpha \in [0; 1]$, whenever choosing $x_C \in [x_A; x_B]$ is optimal. This implies $x_A = 0.25$ and $x_B = 0.75$. This is the case chosen by Prescott and Visscher, who assume moreover $\alpha = 0.50$, which yields *point E*.

Segment G–F gives B a share of 0.25, which, as stressed above, implies $x_A = 0.25$ [since B should get at least $(1 - x_A)/3$]. A point in this segment giving a^* to A can be sustained by the following strategy for C: 'A should choose $x_A = 0.25$, otherwise C goes next to it, whenever choosing $x_C \in [x_A; x_B]$ is optimal; given $x_A = 0.25$, B should choose $x_B = 0.75 + 2 \cdot (0.50 - a^*)$, otherwise C goes next to B (whenever $x_C \in [x_A; x_B]$ is optimal). If A and B collaborate with C, C chooses $x_C = 0.75 - 2 \cdot (0.50 - a^*)$.' In equilibrium, $a = a^*$, and $b = 0.25$, with A and B unable to be better off by not collaborating with C. In this case, the minimum A will get is when $x_B = 1$ and $x_C = 0.50$, which is represented by point G in fig. 1.

Point H is the most favorable for firm C. We have seen it involves ($x_A = 0.10$, $x_B = 1$, $x_C = 0.40$). It can be enforced by the following strategy for C (whenever choosing $x_C \in [x_A; x_B]$ is optimal):

(i) A should go at 0.10 and B at 1. If A does not follow this rule, B should still go at 1. Then C will choose $x_C = x_A$. If $x_B \neq 1$, C chooses $x_C = x_B$. This gives B the incentive to collaborate, and by choosing $x_A \neq 0.10$, A gets at most 0.25 (for $x_A = 0.25$).
(ii) If $x_A = 0.10$, but $x_B \neq 1$, C will choose $x_C = x_B$, yielding a share of at most $(1 - x_A)/3$, $= 0.30$, for B.

By this rule, neither A nor B is better off by not cooperating. Points on the *segment H–G* can be obtained by the same rule but allowing x_A to move gradually to the right, with x_C moving also to the right such as to give B exactly $(1 - x_A)/3$.

Finally, *segment D–H* gives 0.25 to firm A, and can be the result of various rules, since A cannot guarantee itself more by playing individually. Starting from *H*, which we have considered above, C can allow x_B to move to the left, up to the point where $x_B = 0.70$ (i.e., the point where C is just indifferent between $x_C = 0.40$ and locating to the right of B), which leads to the allocation ($a = 0.25$, $b = 0.45$, $c = 0.30$). One can also start from point *D* ($x_A = x_C = 0.25$, $x_B = 0.75$) and move x_B to the right [by C's strategy (when $x_C \in [x_A; x_B]$ is optimal): '$x_C = x_B$, unless B goes at a particular $x_B^* > 0.75$; then $x_C = x_A$']. This can be done until $x_B = 1$, which yields ($a = 0.25$, $b = c = 0.375$). The whole segment *D–H* can thus be sustained.

We have shown how the entire frontier can be sustained. The interior of this set can be sustained by combinations of the various rules described above for C, especially since in these cases A and B get both strictly more than what they can guarantee themselves by playing alone. [2,3] Note that the

[2] We still have to check that there exist locations satisfying $x_A \leq 0.25$, $x_B \geq (2 + x_A)/3$ and $x_C \in [x_A; x_B]$ sustaining such interior points. We show this by 'slicing' *DHGF* into three parts:

- For area *EGF*, start from a point on *GF*. It implies $x_A = 0.25$, $x_B = 0.75 + y$, $x_C = 0.75 - y$, with $0 \leq y \leq 0.25$. One can then move vertically in fig. 1 up to segment *EF* by gradually shifting x_B to the left, up to 0.75.
- For area *DIE*, start from a point on *DI*. It can be sustained by $x_A = x_C = 0.25$, and $x_B = 0.75 + y$, with $0 \leq y \leq 0.25$. One can then move horizontally in fig. 1 by shifting x_C to the right and x_B to the left *with the same intensity*. Segment *DE* is then reached when $x_B = 0.75$ and $x_C = 0.25 + y$.
- Finally, for area *IHGE*, one can start from points on *IH* and *GH* and move linearly to *E* (which implies $x_A = 0.25$, $x_C = 0.50$, $x_B = 0.75$). For example, we have seen that points on *IH* can be sustained with locations $x_A = 0.10$, $x_C = 0.40$, $x_B = 1.00 - y$, with $0 \leq y \leq 0.15$. Starting from there, one moves linearly towards *E* by shifting x_A towards 0.25, x_C towards 0.50 and x_B towards 0.75, with appropriately chosen intensities [e.g., x_A has to move 50% faster than x_C, since $(0.25 - 0.10) = 1.5\ (0.50 - 0.40)$].

The same method can be used when starting from points on *GH*.

[3] The discussion above assumed that the identities of A and B were directly observable by C. In Dewatripont (1986), we show that an outcome arbitrarily close to the point *H* in fig. 1 can still be sustained by firm C when the identity of A and B is not public information (in order to sustain such an outcome, C has to design a strategy which will reveal the identity of A and B through their respective location choice).

points on the segments D–H, H–G and G–F are such that firms A and B are indifferent between cooperating or not. Such indifference is, however, of a totally different nature from the indifference analyzed above: it disappears whenever A and B receive an additional ϵ (arbitrary small) as market share, i.e., whenever we move into the interior of the shaded area. This is not true for C's indifference, which is a 'structural' feature of the model.

2.5. *Significance of the example*

The example discussed in this section is only illustrative. Its assumptions are too restrictive to be useful as a descriptive location model. However, while the number of equilibria will depend crucially on the *amount of indifference* of firm C and thus on particular assumptions of the model, the multiplicity of equilibrium outcomes will remain in numerous cases:

- The model makes the number of entrants exogenous. One way to relax this assumption is by postulating a fixed entry cost. For example, a fixed cost of 0.2 would still lead to three firms in the market in equilibrium. It would also limit the strategic possibilities of firm C: punishing A and B is made more difficult by the necessity to prevent a fourth firm from entering. Inside these limits, the problem is, however, the same as above.
- Price competition can also be introduced, as a second stage of the game. Prescott and Visscher (1977), Lane (1980), and d'Aspremont et al. (1979) provide examples of this as well as numerical computations. Economides (1983, 1984) also presents general results for the case of simultaneous entry, and Eaton and Wooders (1983) do the same for sequential location. In this last case, the indifference of the example above disappears: locating as far away as possible from rivals becomes strongly optimal, in order to limit the intensity of price competition. Through endogenized entry (with fixed costs), the outcome is moreover unique. Note, however, that having price competition alone does not eliminate all indifference problems: if one assumes the number of entrants to be exogenous, the last firm to enter may face several intervals of equal length, and be indifferent between their respective middle points. While the degree of indifference (and thus of 'power') is more limited than in the example above, the possibility of influencing the strategies of the earlier firms remains.

These two remarks concentrated on exogenous assumptions of the model. There is an endogenous result of the model also worth considering: in some of the sustainable equilibria (for example, point H), the latter one enters, the better-off one ends up. The exogenous order of entry assumed by the model then becomes questionable (as stressed by Prescott and Visscher): why could firm A, for example, not wait until someone else enters? One easy and ad hoc way to solve the problem is to assume that entry takes time, and that the high profits made before the next firm enters more than outweigh the subsequent loss in market share due to entering first. A more reasonable treatment would explicitly introduce dynamics and uncertainty about future entry, but is beyond the scope of this note.

3. Discussion

Section 2 has illustrated how indifference significantly expands equilibrium sets in a sequential model of spatial competition. One way to avoid this weakening of the predictive power of the model is to introduce additional features which eliminate indifference. Eaton and Wooders' (1983) model has this property, through the simultaneous introduction of price competition and endogenous entry.

While this approach is certainly viable in some cases, it can be potentially costly in terms of realism, or be impossible, in others. One may thus instead recognize indifference as a given, and concentrate on the choice of a particular equilibrium outcome. Resorting to 'focal points' would be one solution: for example, point E could be a candidate, since it assumes firm C will always choose to be equally spaced between A and B (this was Prescott and Visscher's assumption). However, there is no strong incentive for C to stick to this solution. Ex ante, it would instead like to announce another strategy: from its point of view, point H is the optimum, and it would like A and B to believe that it will follow a strategy which sustains H. Choosing the outcome favorable to the indifferent player can in fact be rationalized by three types of arguments:

- There is an incentive for the indifferent player to find ways to commit ex ante to resolving its ties so as to achieve its preferred equilibrium outcome (note, however, that indifference is not 'far away' from strict preference; in reality, one might see players trying to influence the tastes of the indifferent player).
- Indifference means power, freedom of action. While the argument of subgame-perfection is to rule out Nash equilibria supported by incredible threats, the idea here is to allow for *all* credible ones.
- The models in the spatial competition literature all assume 'sophisticated' players, i.e., players who expect profit maximization from their rivals. This solution means that players expect their rivals to use all possible means to achieve it.

Since the problem of indifference goes beyond spatial competition models, it would be interesting to formalize this selection criterion among equilibria, and to see how it can be generalized to the case where several players show indifference.

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