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Endogenous choice of price or quantity contract and the implications of two-part-tariff in a vertical structure[☆]

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ABSTRACT

We re-investigate the endogenous choice of price (Bertrand) and quantity (Cournot) contract in the presence of a vertically related upstream market for input. We find that choosing price contract is the dominant strategy for downstream firms when the two-part-tariff pricing contract is determined through centralised Nash bargaining. We further show that the level of social welfare is the same regardless of the mode of product market competition (i.e., Bertrand or Cournot).

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1. Introduction

There is a well-established line of research analysing the effects of Bertrand and Cournot competition on profit and social welfare. In a seminal paper, Singh and Vives (1984) show that choosing quantity (price) contract is the dominant strategy for both firms when the goods are substitutes (complements). Furthermore, firms' profits are higher under Cournot competition whereas Bertrand competition yields higher social welfare when the input markets are competitive[1]. However, it is often found that input suppliers and the final goods producers are involved in two-part tariff vertical pricing contracts (Berto Villa-Boas, 2007 and Bonnet and Dubois, 2010). Alipranti et al. (2014) show that when a monopoly input supplier and two final goods producers determine

the two-part tariff vertical pricing contracts through a *decentralised* generalised Nash bargaining process, the equilibrium profits of the final goods producers and social welfare are higher under Cournot competition.

We, on the other hand, aim to revisit the classic question of price and quantity contract where the downstream firms involve in *centralised bargaining*[2] with an upstream input supplier to determine the two-part tariff vertical pricing contracts. We show that choosing price contract is the dominant strategy for the downstream firms and both Bertrand and Cournot entail equal welfare level. López and Naylor (2004), López (2007), Mukherjee et al. (2012) also consider the implications of Bertrand and Cournot competition in isolation, under strategic input-price determination. Their results mostly confirm Singh and Vives' (1984) findings and hence, our results are in stark contrast with the existing literature.

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    E-mail address: d.basak@swansea.ac.uk (D. Basak).
[1] See Delbono and Denicolò (1990), Qiu (1997) and Häckner (2000) for some works on Bertrand and Cournot competition under competitive input markets.

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[2] The implications of centralised bargaining is justifiable in most continental European countries, such as Germany (Hirsch et al., 2014). In the context of strategic input-price determination (Calmfors and Driffill, 1988; Danthine and Hunt, 1994) argue that collective bargaining is more widely accepted as it internalises various negative externalities, such as unemployment. Also, see Flanagan (2003), Boeri and Burda (2009) for a critical survey on this strand of literature.

2. The model

We consider an economy with two downstream firms, denoted by D_i producing differentiated products where $i, j = 1, 2$ and $i \neq j$. The downstream firms require a critical input for production that they purchase from a monopoly input supplier, U, through two-part tariff contracts involving an up-front fixed-fee and a per-unit price. U produces the inputs at a constant marginal cost of production, c which we assume to be zero. We assume that one unit of input is required to produce one unit of the output, and D_i and D_j can convert the inputs to the final goods without incurring any further cost.

We develop a model of three stage game. At stage 1, each downstream firm simultaneously chooses whether to adopt *quantity contract* or *price contract*. At stage 2, U is involved in a centralised bargaining with a representative of D_1 and D_2 to determine the terms of the two-part tariff contracts involving an up-front fixed-fee, F_i, and a per-unit price, w_i, $i = 1, 2$. At stage 3, firms compete contingent to the decisions made in stage 1. We solve the game through backward induction.

Hence, we start our discussion at stage 3. To this extent we consider four possible constellations, $\{\rho = qq, pp, pq, qp\}$; that attribute to the following properties:

- ($\rho = qq$): where both firms adopt quantity contracts
- ($\rho = pp$): where both firms adopt price contracts
- ($\rho = pq$): where D_1 chooses price contract and D_2 chooses quantity contract
- ($\rho = qp$): where D_1 chooses quantity contract and D_2 chooses price contract.

We work out the equilibrium outcomes under each of these strategy combinations.

At stage 2 U, the monopoly input supplier and a representative of D_1 and D_2 determine the terms of the two-part tariff contract by maximising the following generalised Nash bargaining expression

$$\underset{F_i^\rho, w_i^\rho}{\text{Max}} \left[\sum_{i=1}^{2} \left(w_i^\rho q_i^\rho + F_i^\rho \right) \right]^\beta \left[\sum_{i=1}^{2} \left(\pi_i^\rho - F_i^\rho \right) \right]^{1-\beta} \quad (1)$$

where q_i^ρ and $\left(\pi_i^\rho - F_i^\rho \right)$ denote the output and net profit of the downstream firms and β (resp. $(1 - \beta)$) shows the bargaining power of the input supplier (resp. final goods producers). We restrict our analysis to $\beta \in (0, 1)$.

Maximising the above with respect to F_i gives the following[3]

$$F_i^\rho = \frac{1}{2} \left[\beta \sum_{i=1}^{2} \pi_i^\rho - (1 - \beta) \sum_{i=1}^{2} w_i^\rho q_i^\rho \right]. \quad (2)$$

Substituting (2) in (1), we get the maximisation problem as

$$\underset{w_i^\rho}{\text{Max}} \left[\beta \sum_{i=1}^{2} \left(\pi_i^\rho + w_i^\rho q_i^\rho \right) \right]^\beta \left[(1 - \beta) \sum_{i=1}^{2} \left(\pi_i^\rho + w_i^\rho q_i^\rho \right) \right]^{1-\beta}. \quad (3)$$

Eq. (3) shows that the per-unit input price is determined to maximise the industry profit (i.e., the total profits of U, D_1 and D_2), since

$$\sum_{i=1}^{2} \left(\pi_i^\rho + w_i^\rho q_i^\rho \right) = \sum_{i=1}^{2} \left[\left(P_i^\rho - w_i^\rho \right) q_i^\rho + w_i^\rho q_i^\rho \right]$$

$$= \sum_{i=1}^{2} P_i^\rho q_i^\rho$$

which is the profit of a monopoly final goods producer, producing both the products at zero marginal cost of production. Hence, it is intuitive that the centralised bargaining entails same level of output, consumer surplus and social welfare irrespective of the mode of the contract (i.e., Bertrand or Cournot) chosen by the final goods producers.

Corollary 1. *If the final goods producers and a monopoly input supplier involve in centralised generalised Nash bargaining to determine the two-part tariff vertical pricing contract, the outcomes yield equal level of output, consumer surplus and social welfare irrespective of the type of the product market competition.*

3. Equilibrium outcomes

We now find out the equilibrium outcomes under a specific (inverse) demand function: $P_i = 1 - q_i - \gamma q_j$, similar to Alipranti et al. (2014) with an exception that we normalise the demand intercept to unity for simplicity. $\gamma \in (0, 1)$ measures the degree of product differentiation. If $\gamma = 1$, the goods are perfect substitutes, and if $\gamma = 0$, the goods are isolated.

3.1. (q–q) contract

Downstream firm's profit motive yields

$$\underset{q_i}{\text{Max}} \, D\pi_i^{qq} = \pi_i^{qq} - F_i$$

$$= \left(1 - q_i - \gamma q_j - w_i \right) q_i - F_i. \quad (4)$$

Solving the first order conditions we obtain the equilibrium output of the ith firm

$$q_i^{qq} = \frac{(2 - \gamma) - 2w_i + \gamma w_j}{4 - \gamma^2}. \quad (5)$$

Given (5), the profit equation in (4) reduces to

$$D\pi_i^{qq} = (q_i)^2 - F_i. \quad (6)$$

Maximising (3) subject to (5) and (6) gives the equilibrium per-unit input price and upfront fixed fee as

$$w_i^{qq} = \frac{1}{2} \left(\frac{\gamma}{1 + \gamma} \right).$$

Given the per-unit input price, the equilibrium fixed-fees gives

$$F_i^{qq} = \frac{(\beta - \gamma + \beta\gamma)}{4(1 + \gamma)^2}.$$

The equilibrium downstream profit is

$$D\pi_i^{qq} = \frac{1}{4} \left(\frac{1 - \beta}{1 + \gamma} \right). \quad (7)$$

Next, we work out the consumer surplus and social welfare, are respectively as

$$CS^{qq} = \frac{1}{4} \left(\frac{1}{1 + \gamma} \right) \quad \text{and} \quad SW^{qq} = \frac{3}{4} \left(\frac{1}{1 + \gamma} \right). \quad (8)$$

3.2. (p–p) contract

In order to solve the Bertrand game we derive the direct demand function $q_i = \frac{(1-\gamma) - P_i + \gamma P_j}{1 - \gamma^2}$. Accordingly, the representative

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[3] Like Alipranti et al. (2014) we also allow negative fixed-fees, which occurs for a small β. The upstream firm, in this case, subsidises downstream's production via fixed-fee.

downstream firm maximises the following

$$\underset{P_i}{\text{Max}} \; D\pi_i^{pp} = \pi_i^{pp} - F_i$$

$$= (P_i - w_i) \left(\frac{(1-\gamma) - P_i + \gamma P_j}{1 - \gamma^2} \right) - F_i. \qquad (9)$$

The equilibrium price and output of the ith firm can be found as

$$P_i^{pp} = \frac{(1-\gamma)(2+\gamma) + 2w_i + \gamma w_j}{4 - \gamma^2}$$

$$q_i^{pp} = \frac{(1-\gamma)(2+\gamma) - (2 - \gamma^2) w_i + \gamma w_j}{(4 - \gamma^2)(1 - \gamma^2)}. \qquad (10)$$

Given the above, downstream's profit maximisation problem in (9) reads as

$$D\pi_i^{pp} = (1 - \gamma^2)(q_i)^2 - F_i. \qquad (11)$$

Maximising (3) subject to (9) and (10) gives the equilibrium per-unit input price and fixed fees

$$w_i^{pp} = \frac{\gamma}{2} \quad \text{and} \quad F_i^{pp} = \frac{1}{4}\left(\frac{\beta - \gamma}{1 + \gamma} \right).$$

The net equilibrium profits of D_1 and D_2 are

$$D\pi_i^{pp} = \frac{1}{4}\left(\frac{1 - \beta}{1 + \gamma} \right). \qquad (12)$$

We find the consumer surplus and social welfare as below

$$CS^{pp} = \frac{1}{4}\left(\frac{1}{1+\gamma} \right) \quad \text{and} \quad SW^{pp} = \frac{3}{4}\left(\frac{1}{1+\gamma} \right). \qquad (13)$$

3.3. (p–q) contract

Next consider the situation where D_1 chooses the price contract and D_2 chooses the quantity contract. The maximisation problem of the downstream firms yield

$$\underset{P_1}{\text{Max}} \; D\pi_1^{pq} = \pi_1^{pq} - F_1$$

$$= (P_1 - w_1)(1 - P_1 - \gamma q_2) - F_1 \qquad (14a)$$

and

$$\underset{q_2}{\text{Max}} \; D\pi_2^{pq} = \pi_2^{pq} - F_2$$

$$= (1 - q_2 - \gamma(1 - P_1 - \gamma q_2) - w_2) q_2 - F_2. \qquad (14b)$$

Maximising (14a) and (14b) determines the equilibrium price charged by D_1 and the corresponding output level of D_2 respectively.

$$P_1^{pq} = \frac{(1-\gamma)(2+\gamma) + 2(1 - \gamma^2) w_1 + \gamma w_2}{4 - 3\gamma^2}$$

$$q_2^{pq} = \frac{(2-\gamma) + \gamma w_1 - 2w_2}{4 - 3\gamma^2}. \qquad (15)$$

Hence, the profit equations in (14) and (15) reduce to

$$D\pi_1^{pq} = (q_1)^2 - F_1 \qquad (16a)$$

$$D\pi_2^{pq} = (1 - \gamma^2)(q_2)^2 - F_2. \qquad (16b)$$

We derive the per-unit input price and upfront fixed fees by generalised Nash bargaining

$$w_1^{pq} = \frac{1}{2}\left(\frac{\gamma}{1+\gamma} \right) \quad \text{and} \quad w_2^{pq} = \frac{\gamma}{2}$$

and

$$F_1^{pq} = F_2^{pq} = \frac{2\beta - 2\gamma + 2\beta\gamma - \gamma^2}{8(1+\gamma)^2}.$$

The net equilibrium profits of D_1 and D_2 are

$$D\pi_1^{pq} = \frac{2 - 2\beta + 2\gamma - 2\beta\gamma + \gamma^2}{8(1+\gamma)^2} \quad \text{and}$$

$$D\pi_2^{pq} = \frac{2 - 2\beta + 2\gamma - 2\beta\gamma - \gamma^2}{8(1+\gamma)^2}. \qquad (17)$$

The equilibrium consumer surplus and social welfare are respectively

$$CS^{pq} = \frac{1}{4}\left(\frac{1}{1+\gamma} \right) \quad \text{and} \quad SW^{pq} = \frac{3}{4}\left(\frac{1}{1+\gamma} \right). \qquad (18)$$

3.4. (q–p) contract

Now consider the case where D_1 chooses the quantity contract and D_2 chooses the price contract. The maximisation problem of the downstream firms yield

$$\underset{q_1}{\text{Max}} \; D\pi_1^{qp} = \pi_1^{qp} - F_1$$

$$= (1 - q_1 - \gamma(1 - P_2 - \gamma q_1) - w_1) q_1 - F_1 \qquad (19a)$$

and

$$\underset{P_2}{\text{Max}} \; D\pi_2^{qp} = \pi_2^{qp} - F_2$$

$$= (P_2 - w_2)(1 - P_2 - \gamma q_1) - F_2. \qquad (19b)$$

Maximising (19a) and (19b) and solving the first order conditions give the equilibrium price and quantity of D_1 and D_2 respectively.

$$q_1^{qp} = \frac{(2-\gamma) - 2w_1 + \gamma w_2}{4 - 3\gamma^2}$$

$$P_2^{qp} = \frac{(1-\gamma)(2+\gamma) + \gamma w_1 + 2(1 - \gamma^2) w_2}{4 - 3\gamma^2}. \qquad (20)$$

Hence, the profit equations in (19) give

$$D\pi_1^{qp} = (1 - \gamma^2)(q_1)^2 - F_1 \qquad (21a)$$

$$D\pi_2^{qp} = (q_2)^2 - F_2 \qquad (21b)$$

and the following outcomes

$$w_1^{qp} = \frac{\gamma}{2} \quad \text{and} \quad w_2^{qp} = \frac{1}{2}\left(\frac{\gamma}{1+\gamma} \right)$$

$$F_1^{qp} = F_2^{qp} = \frac{2\beta - 2\gamma + 2\beta\gamma - \gamma^2}{8(1+\gamma)^2}.$$

The net equilibrium profits of D_1 and D_2 are

$$D\pi_1^{qp} = \frac{2 - 2\beta + 2\gamma - 2\beta\gamma - \gamma^2}{8(1+\gamma)^2} \quad \text{and}$$

$$D\pi_2^{qp} = \frac{2 - 2\beta + 2\gamma - 2\beta\gamma + \gamma^2}{8(1+\gamma)^2}. \qquad (22)$$

The equilibrium consumer surplus and social welfare are respectively

$$CS^{qp} = \frac{1}{4}\left(\frac{1}{1+\gamma} \right) \quad \text{and} \quad SW^{qp} = \frac{3}{4}\left(\frac{1}{1+\gamma} \right). \qquad (23)$$

Table 1
First-stage game.

	Quantity	Price
Quantity	$D\pi_1^{qq}, D\pi_2^{qq}$	$D\pi_1^{qp}, D\pi_2^{qp}$
Price	$D\pi_1^{pq}, D\pi_2^{pq}$	$D\pi_1^{pp}, D\pi_2^{pp}$

4. Results

We now analyse the first stage of the game where the downstream firms decide whether to choose *price contract* or *quantity contract*. Table 1 summarises the possible strategies of each firm and the realised profit under the respective scenarios.

The closed form solutions of firms' pay-offs under $\{\rho = qq, pp, pq, qp\}$ are reported in Eqs. (7), (12), (17) and (22) respectively. Straightforward calculations give the following.

$$D\pi_1^{qq} - D\pi_1^{pq} = D\pi_1^{qp} - D\pi_1^{pp} = D\pi_2^{qq} - D\pi_2^{qp}$$
$$= D\pi_2^{pq} - D\pi_2^{pp} = -\frac{1}{8}\left(\frac{\gamma}{1+\gamma}\right)^2 < 0.$$

The proposition below is immediate from the above.

Proposition 1. *Assume that* $\gamma \in (0, 1)$*, choosing price contract is the dominant strategy for both firms.*

The intuition goes as follows. Each final goods producer's profit motive is driven by the amount of input price and upfront fixed fee payable to the upstream agent. First, assume that firm 2 (firm 1) chooses the quantity contract. When β is significantly high, meaning that the input supplier's opportunistic behaviour is more pronounced; firm 1's (firm 2's) output loss following an increase in its own input price is larger under quantity contract than under price contract. Furthermore, the fixed fee being higher under quantity contract[4]; $F_1^{qq} > F_1^{pq}$ $\left(F_2^{qq} > F_2^{qp}\right)$, firm 1 (firm 2) finds it profitable to choose price contract. Next, if firm 2 (firm 1) chooses a price contract firm 1 (firm 2) again prefers a price contract over a quantity contract as the latter generates a greater loss in its own output level and it involves a higher fixed fee; $F_1^{qp} > F_1^{pp}$ $\left(F_2^{pq} > F_2^{pp}\right)$. Hence, choosing price contract becomes the dominant strategy for both downstream firms. However, when β is significantly small, the input supplier offers a lump-sum subsidy to the downstream firms (see *footnote 3*). The

opportunistic behaviour being less significant, in this case, the downstream firms only combat the output reducing effect by choosing price contract (as alluded above).

5. Conclusion

Allowing a centralised generalised Nash bargaining between the input supplier and the final goods producers, we show the social welfare are the same under Bertrand and Cournot competition. Our result that adopting price contract is the dominant strategy for downstream duopoly attracts renewed interest in the bargaining literature.

References

Alipranti, M., Milliou, C., Petrakis, E., 2014. Price vs. quantity competition in a vertically related market. Econom. Lett. 124, 122–126.

Berto Villa-Boas, S., 2007. Vertical relationships between manufacturers and retailers: inference with limited data. Rev. Econom. Stud. 74, 625–652.

Boeri, T., Burda, M.C., 2009. Preferences for collective versus individualised wage setting. Econ. J. 119, 144063.

Bonnet, C., Dubois, P., 2010. Inference on vertical contracts between manufacturers and retailers allowing for nonlinear pricing and resale price maintenance. Rand J. Econ. 41, 139–164.

Calmfors, L., Driffill, J., 1988. Bargaining structure, corporatism and macroeconomic performance. Econ. Policy 3, 13–61.

Danthine, J., Hunt, J., 1994. Wage bargaining structure, employment and economic integration. Econ. J. 104, 528–541.

Delbono, F., Denicolò, V., 1990. R&D investment in a symmetric and homogeneous oligopoly. Int. J. Ind. Organiz. 8, 297–313.

Flanagan, R.J., 2003. Collective bargaining and macroeconomic performance. In: Addison, J.T., Schnabel, C. (Eds.), International Handbook of Trade Unions. Edward Elgar, Cheltenham, pp. 172–196.

Häckner, J., 2000. A note on price and quantity competition in differentiated oligopolies. J. Econom. Theory 93, 233–239.

Hirsch, B., Merkl, C., Müller, S., Schnabel, C., 2014. Centralized vs. decentralized wage formation: The role of firms' production technoloy, IZA Discussion Paper No. 8242.

López, M.C., 2007. Price and quantity competition in a differentiated duopoly with upstream suppliers. J. Econ. Manage. Strategy 16, 469–505.

López, M.C., Naylor, R.A., 2004. The Cournot–Bertrand profit differential: a reversal result in a differentiated duopoly with wage bargaining. Eur. Econ. Rev. 48, 681–696.

Mukherjee, A., Broll, U., Mukherjee, S., 2012. Bertrand versus Cournot competition in a vertical structure: a note. Manchester Sch. 80, 545–559.

Qiu, L.D., 1997. On the dynamic efficiency of Bertrand and Cournot equilibria. J. Econom. Theory 75, 213–229.

Singh, N., Vives, X., 1984. Price and quantity competition in a differentiated duopoly. Rand J. Econ. 15, 546–554.

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[4] Check that $F_1^{qq} - F_1^{pq} = F_1^{qp} - F_1^{pp} = F_2^{qq} - F_2^{qp} = F_2^{pq} - F_2^{pp} = \frac{1}{8}\left(\frac{\gamma}{1+\gamma}\right)^2 > 0.$