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On the Diffusion of New Technology: A Game Theoretic Approach

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1. INTRODUCTION

While much has been written recently on the economics of research and development, most theoretical studies have focused on investment in research when the feasibility and profitability of the innovation may be stochastic (for references see Reinganum (1980*a*, 1980*b*). These authors are especially concerned with the relationship between market structure and the pace of inventive activity. However, the perfection of a new and superior technology confers neither private nor social benefit until that technology is adopted and employed by potential users. In an industry with substantial entry costs, perfection and adoption of an innovation are not necessarily conterminous. While there are many industry-specific and innovation-specific case studies of the diffusion of new technology, the theoretical literature is extremely sparse.

This paper is an attempt at a rigorous (albeit not exceedingly general) analysis of the diffusion of new technology. In particular, consider an industry composed of two firms, each using the current best-practice technology. The firms are assumed to be operating at Nash equilibrium output levels, generating a market price (given demand) and profit allocations. When a cost-reducing innovation is announced, each firm must determine when (if ever) to adopt it, based in part upon the discounted cost of implementing the new technology, and in part upon the behavior of the rival firm. If either firm adopts before the other, it can expect to make substantial profits at the expense of the other firm. On the other hand, the discounted sum of purchase price and adjustment costs may decline with the lengthening of the adjustment period as various quasi-fixed factors become more easily variable. Therefore, although waiting costs the firm more in terms of foregone profits, it may save money on the cost of purchasing the new technology. Thus the firm must weigh the costs and benefits of delaying adoption, as well as take account of its rival's strategic behavior. This problem is formalized and discussed in Section 2. In Section 3 it is shown that there exist two asymmetric Nash equilibria in pure strategies. That is, at a Nash equilibrium, one firm will adopt the innovation at a relatively early date, the other relatively later. Thus even in the case of identical firms and complete certainty, there is a "diffusion" of innovation over time. Section 4 extends the analysis to the case of non-identical firms. In this case, it is shown that a Nash equilibrium exists and is asymmetric. The basic model, though independently developed, closely parallels that of Flaherty (1980). For a discussion of the relationship between this work and that of Flaherty, see Section 4. Section 5 concludes the paper.

2. PROBLEM FORMULATION

Consider a pair of identical firms engaged in a game of duopoly. They are making Nash equilibrium profits[1] of (π_0, π_0) per period. At $t = 0$ a technological improvement designed to reduce production costs is announced and offered for sale.

If firm 1 purchases the new technology before firm 2, then the profit allocations are (π_1, π_2) during each period before firm 2 adopts the innovation. Similarly, if firm 2 adopts first, then the profits are (π_2, π_1) per period until firm 1 adopts. After both firms have adopted the innovation, profits are (π_3, π_3). The industry environment is assumed stationary except for the introduction of the new technology, and no further technical advance is anticipated. For an analysis of a single firm's reaction to anticipated technical advance, see Kamien and Schwartz (1972).

Letting T_1 and T_2 denote the adoption dates of firms 1 and 2, respectively, we can summarize the profit opportunities described above in tabular form.

t	Player 1	Player 2
$0 \leqq t \leqq \min\{T_1, T_2\}$	π_0	π_0
$T_1 \leqq t \leqq T_2$	π_1	π_2
$T_2 \leqq t \leqq T_1$	π_2	π_1
$\infty > t \geqq \max\{T_1, T_2\}$	π_3	π_3

The following assumptions describe the relative magnitudes of the profit rates.

Assumption 1. $\pi_i > 0 \; \forall i \in \{0, 1, 2, 3\}$.

Assumption 2. $\pi_1 > \pi_3 > \pi_2$; $\pi_1 > \pi_0 > \pi_2$.

Assumption 3. $\alpha = \pi_1 - \pi_0 + \pi_2 - \pi_3 > 0$.

Assumption 1 states that in any event, both firms make positive profits. Assumption 2 implies that profits to firm i are greatest when i has adopted the innovation but j has not; next greatest profits occur either when both have adopted or when no firm has yet adopted; finally profit opportunities for i are worst when j has adopted the innovation but i has not. Assumption 3 states that the increment to revenue when one is first exceeds the increment to revenue when one is second. Alternatively, the net value of being first, $\alpha = (\pi_1 - \pi_0) - (\pi_3 - \pi_2)$, is positive.[2]

Define $p(t)$ to be the discounted price of the innovation at t. This may be assumed to include all relevant costs of adjustment. Assume that this function exists and is C^2 for all $t \in [0, \infty)$, though purchases may take place at selected dates rather than continuously.

Let r represent the market rate of interest. The firms' payoffs are given in Definition 1 below.

Definition 1. The payoff to firm 1 is

$$V^1(T_1, T_2) \equiv \begin{cases} g^1(T_1, T_2) & \text{if } T_1 \leqq T_2 \\ g^2(T_1, T_2) & \text{if } T_1 \geqq T_2 \end{cases}$$

where

$$g^1(T_1, T_2) = \int_0^{T_1} \pi_0 e^{-rt} dt + \int_{T_1}^{T_2} \pi_1 e^{-rt} dt + \int_{T_2}^{\infty} \pi_3 e^{-rt} dt - p(T_1)$$

and

$$g^2(T_1, T_2) = \int_0^{T_2} \pi_0 e^{-rt} dt + \int_{T_2}^{T_1} \pi_2 e^{-rt} dt + \int_{T_1}^{\infty} \pi_3 e^{-rt} dt - p(T_1).$$

The payoff to firm 2 is simply $V^2(T_1, T_2) \equiv V^1(T_2, T_1)$. To avoid confusion (and without loss of generality) I will deal almost exclusively with firm 1's decision problem. The corresponding results for firm 2 can be deduced by symmetry.

Notice that while $V^1(T_1, T_2)$ is continuous in T_1 for fixed T_2, it is not differentiable at $T_1 = T_2$. The left-hand derivative at T_2 is

$$g_1^1(T_2, T_2) = (\pi_0 - \pi_1) e^{-rT_2} - p'(T_2)$$

while the right-hand derivative at T_2 is

$$g_1^2(T_2, T_2) = (\pi_2 - \pi_3) e^{-rT_2} - p'(T_2).$$

By Assumption 3, $g_1^1(T_2, T_2) < g_1^2(T_2, T_2)\ \forall T_2 \in [0, \infty)$.

Assumption 4. Assume $p(t) \geqq 0$ and $p''(t) > r[\pi_1 - \pi_0]e^{-rt}\ \forall t \in [0, \infty)$. Further assume that $\lim_{t\to\infty} p'(t) > 0$.

Since $p(t)$ represents the present value cost of implementing the new technology over the adjustment period t, Assumption 4 states that cost saving can't continue forever. Rather, there is some optimal adjustment period beyond which any further prolongation of the adjustment process begins to increase costs. Without this assumption, firms may postpone adoption forever. Since one is primarily interested in the timing of adoption by strategic players (rather than circumstances under which nonadoption is optimal), Assumption 4 will be maintained throughout. Assumptions 1–3 will also be maintained throughout. One of the versions of Assumption 5 below will be specified as a hypothesis in the results which follow.

Assumption 5a. $p'(0) < \pi_2 - \pi_3$.

Assumption 5b. $p'(0) \geqq \pi_2 - \pi_3$.

Under Assumption 4 it is easy to show that $g^1(T_1, T_2)$ and $g^2(T_1, T_2)$ are strictly concave[3] in T_1 (for fixed T_2). To see this, note that $g_{11}^1 = r(\pi_1 - \pi_0) e^{-rt} - p''(t) < 0$ by Assumption 4. Since $g_{11}^2 = r(\pi_3 - \pi_2) e^{-rt} - p''(t)$and $\pi_3 - \pi_2 < \pi_1 - \pi_0$ by Assumption 3, $g_{11}^2 < 0$. In addition, one can prove the following preliminary lemmas.

Lemma 1. $\exists$ *unique* $\hat{T} \in [0, \infty)$ *and* $\hat{\hat{T}} \in [0, \infty)$ *which maximize* $g^1(T_1, T_2)$ *and* $g^2(T_1, T_2)$, *respectively, independent of* T_2. *Furthermore*

(a) *if Assumption* 5*a holds, then* $0 \leqq \hat{T} < \infty$ *and* $0 < \hat{\hat{T}} < \infty$.

(b) *if Assumption* 5*b holds, then* $\hat{T} = \hat{\hat{T}} = 0$.

Proof. $\forall T_2[\lim_{t\to\infty} g_1^1(t, T_2) = \lim_{t\to\infty} (\pi_0 - \pi_1) e^{-rt} - p'(t) < 0]$ by Assumption 4.[4] By strict concavity and continuity of g^1, there exists a unique $\hat{T} \in [0, \infty)$ which maximizes $g^1(T_1, T_2)$. $\hat{T}$ is defined (independent of T_2) by

$$(\pi_0 - \pi_1) e^{-r\hat{T}} - p'(\hat{T}) \leqq 0, \quad \hat{T} \geqq 0, \quad \text{and} \quad [(\pi_0 - \pi_1) e^{-r\hat{T}} - p'(\hat{T})]\hat{T} = 0. \tag{3}$$

If Assumption 5b holds, then $\forall T_2[g_1^1(0, T_2) < 0]$, so $\hat{T} = 0$.

$\forall T_2[\lim_{t\to\infty} g_1^2(t, T_2) = \lim_{t\to\infty} (\pi_2 - \pi_3) e^{-rt} - p'(t) < 0]$ by Assumption 4.

By strict concavity and continuity of g^2, there exists a unique $\hat{\hat{T}} \in [0, \infty)$ which maximizes $g^2(T_1, T_2)$. $\hat{\hat{T}}$ is defined (independent of T_2) by

$$(\pi_2 - \pi_3)\, e^{-r\hat{\hat{T}}} - p'(\hat{\hat{T}}) \leqq 0, \quad \hat{\hat{T}} \geqq 0, \quad \text{and} \quad [(\pi_2 - \pi_3)\, e^{-r\hat{\hat{T}}} - p'(\hat{\hat{T}})]\hat{\hat{T}} = 0. \tag{4}$$

If Assumption 5a holds, then $\forall T_2[g_1^2(0, T_2) = (\pi_2 - \pi_3) - p'(0) > 0]$, so $\hat{\hat{T}} > 0$. If Assumption 5b holds, then $\forall T_2[g_1^2(0, T_2) = (\pi_2 - \pi_3) - p'(0) \leqq 0]$, and strict concavity ($g_{11}^2 < 0$) implies that $\hat{\hat{T}} = 0$. ||

Condition (3) states that if the first adopter is to adopt after some delay ($\hat{T} > 0$), then the marginal (discounted) opportunity cost of waiting, $(\pi_1 - \pi_0)e^{-r\hat{T}}$, must be exactly off-set by the marginal (discounted) cost savings due to waiting, $-p'(\hat{T})$. If the marginal (discounted) opportunity cost of waiting exceeds the marginal (discounted) cost savings for all T, then immediate adoption is optimal ($\hat{T} = 0$). Condition (4) has a similar interpretation for the second adopter.

Lemma 2. *If Assumption 5a holds, then* $\hat{\hat{T}} > \hat{T}$.

Proof. Note that $\forall T_2[g_1^2(t, T_2) > g_1^1(t, T_2) \forall t \in [0, \infty)]$. By Lemma 1, $\infty > \hat{\hat{T}} > 0$ and $\hat{T} < \infty$. Thus the corner case is obvious: if $\hat{T} = 0$, then $\hat{\hat{T}} > 0$ implies $\hat{\hat{T}} > \hat{T}$. Suppose $\hat{T} > 0$. Now $\forall T_2[(g_1^2(\hat{T}, T_2) > g_1^1(\hat{T}, T_2) = 0]$. Since $g_1^2(\hat{\hat{T}}, T_2) = 0$ and $g^2(\cdot, T_2)$ is strictly concave, $\hat{\hat{T}} > \hat{T}$. ||

3. NASH EQUILIBRIUM IN PURE STRATEGIES

The problem set out in Section 2 (that of determining an optimal adoption date when a competitor exists) can be modeled and solved in a game theoretic framework.

Definition 2. The strategy space for player i is $S_i = [0, \infty)$. A pure strategy for i is a scalar $T_i \in S_i$.

Definition 3. The set of best responses for i to T_j is $\phi_i(T_j) = \{T_i \in S_i \mid V^i(T_i, T_j) \geqq V^i(T_i', T_j)\ \forall T_i' \in S_i\}$.

The mapping $\phi_i : S_j \Rightarrow S_i$ is i's best response correspondence.

Definition 4. A strategy pair (T_1^N, T_2^N) is a Nash equilibrium for the game $G = (V^1, V^2, S^1, S^2)$ if

(a) $T_i^N \in S_i$, $i = 1, 2$;

(b) $V^1(T_1^N, T_2^N) \geqq V^1(T_1, T_2^N)\ \forall T_1 \in S_1$; and

(c) $V^2(T_1^N, T_2^N) \geqq V^2(T_1^N, T_2)\ \forall T_2 \in S_2$.

Alternatively, the pair (T_1^N, T_2^N) is a Nash equilibrium if $T_1^N \in \phi_1(T_2^N)$ and $T_2^N \in \phi_2(T_1^N)$; that is, each strategy is a best response to the other.

Theorem 1.

(a) *If Assumption 5a holds, then there exist 2 Nash equilibria in pure strategies*[5] $(T_1^N, T_2^N) = (\hat{T}, \hat{\hat{T}})$ and $(T_1^N, T_2^N) = (\hat{\hat{T}}, \hat{T})$.

(b) *If Assumption 5b holds, then there exists a unique* $(T_1^N, T_2^N) = (0, 0)$.

Proof

(a) See below

(b) It is clear from Lemma 1 that immediate adoption is a dominant strategy for each firm. ||

Before proceeding to prove part (a), consider the implications of Theorem 1. There is the degenerate case (b). In this case, adjustment costs do not decline sufficiently rapidly as to warrant waiting. Thus, regardless of rival behavior, the best action a firm can take is to adopt immediately.

Excepting this degenerate case, we see that, if there is a net value to being first (Assumption 3), then at a Nash equilibrium one firm adopts early (at $\hat{T}$), while the other adopts late at $\hat{\hat{T}}$). It is *never* a Nash equilibrium for identical firms to bring the new technology on line at the same date. Hence even in the case of identical firms and complete certainty, there will be a diffusion of the innovation over time.

The proof of Theorem 1(a) is accomplished via a series of lemmas.

Lemma 3. *$g^1(T_1, T_2) \gtreqless g^2(T_1, T_2)$ as $T_1 \lesseqgtr T_2$.*

Proof. $g^1(T_1, T_2) - g^2(T_1, T_2) = (\alpha/r)(e^{-rT_1} - e^{-rT_2}) \gtreqless 0$ as $T_1 \lesseqgtr T_2$. ||

Lemma 4. $g^2(\hat{\hat{T}}, \hat{T}) > g^1(\hat{T}, \hat{T})$.

Proof. $g^2(\hat{\hat{T}}, \hat{T}) > g^2(\hat{T}, \hat{T}) = g^1(\hat{T}, \hat{T})$, where the inequality follows from Lemma 1. The equality follows from Lemma 3. ||

Lemma 5. $g^1(\hat{T}, \hat{\hat{T}}) > g^2(\hat{\hat{T}}, \hat{\hat{T}})$.

Proof. $g^1(\hat{T}, \hat{\hat{T}}) > g^1(\hat{\hat{T}}, \hat{\hat{T}}) = g^2(\hat{\hat{T}}, \hat{\hat{T}})$, where the inequality follows from Lemma 1. The equality follows from Lemma 3. ||

Lemma 6. *$\exists \tilde{T} \in (\hat{T}, \hat{\hat{T}})$ such that $g^1(\hat{T}, T_2) \lesseqgtr g^2(\hat{\hat{T}}, T_2)$ as $T_2 \lesseqgtr \tilde{T}$.*

Proof. Let $\gamma(\hat{T}, \hat{\hat{T}}, T_2) = g^1(\hat{T}, T_2) - g^2(\hat{\hat{T}}, T_2)$. By Lemma 4, $\gamma(\hat{T}, \hat{\hat{T}}, \hat{T}) < 0$. By Lemma 5, $\gamma(\hat{T}, \hat{\hat{T}}, \hat{\hat{T}}) > 0$. Since $\partial\gamma/\partial T_2 = \alpha\, e^{-rT_2} > 0$, it follows by the intermediate value theorem and the monotonicity of γ in T_2 that $\exists$ a unique $\tilde{T} \in (\hat{T}, \hat{\hat{T}})$ such that $\gamma(\hat{T}, \hat{\hat{T}}, T_2) \lesseqgtr 0$ as $T_2 \lesseqgtr \tilde{T}$. ||

Figures 1–3 illustrate Lemmas 3–6.

Lemma 7.

$$\phi_1(T_2) = \begin{cases} \hat{\hat{T}} & \text{for } T_2 < \tilde{T} \\ \{\hat{T}, \hat{\hat{T}}\} & \text{for } T_2 = \tilde{T} \\ \hat{T} & \text{for } T_2 > \tilde{T} \end{cases}$$

Proof. Recall Definition 1

$$V^1(T_1, T_2) = \begin{cases} g^1(T_1, T_2) & T_1 \leqq T_2 \\ g^2(T_1, T_2) & T_1 \geqq T_2 \end{cases}.$$

The reasons for the various relations are found in parentheses immediately below the relation itself.

Case 1. $T_2 < \tilde{T}$.

$$\forall T_1 \leqq T_2 [\underset{\text{(Def. 1)}}{V^1(\hat{\hat{T}}, T_2) = g^2(\hat{\hat{T}}, T_2)} \underset{\text{(Lemma 6)}}{>} g^1(\hat{T}, T_2) \underset{\text{(Lemma 1)}}{\geqq} g^1(T_1, T_2) \underset{\text{(Def. 1)}}{=} V^1(T_1, T_2)]$$

$$\forall T_1 \geqq T_2, T_1 \neq \hat{\hat{T}} [V^1(\hat{\hat{T}}, T_2) \underset{\text{(Def. 1)}}{=} g^2(\hat{\hat{T}}, T_2) \underset{\text{(Lemma 1)}}{>} g^2(T_1, T_2) \underset{\text{(Def. 1)}}{=} V^1(T_1, T_2)].$$

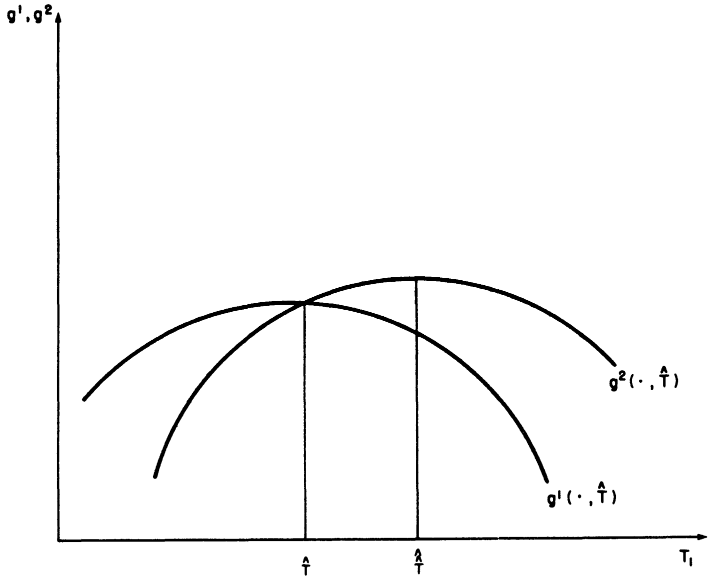

FIGURE 1

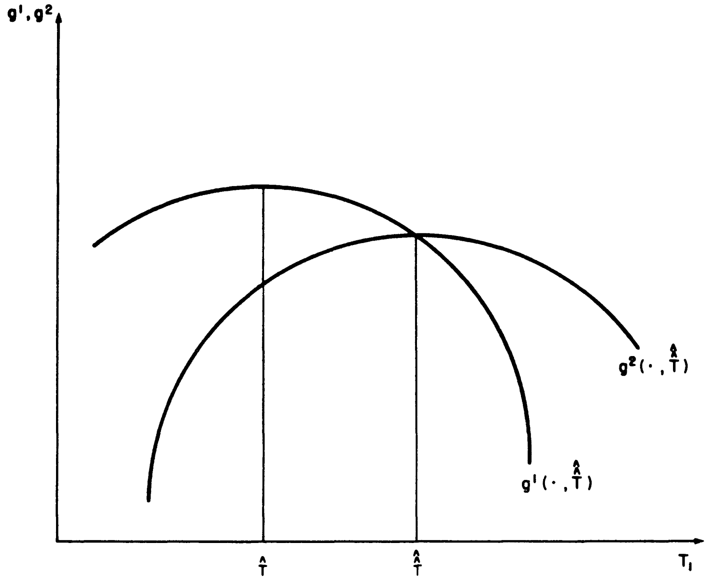

FIGURE 2

Thus $\forall T_1 \neq \hat{\hat{T}}[V^1(\hat{\hat{T}}, T_2) > V^1(T_1, T_2)]$. Hence $\phi_1(T_2) = \hat{\hat{T}}$ for all $T_2 < \tilde{T}$.

Case 2. $T_2 = \tilde{T}$.

$$\forall T_1 \leqq \tilde{T},\ T_1 \neq \hat{T}[V^1(\hat{T}, \tilde{T}) \underset{\text{(Def. 1)}}{=} g^1(\hat{T}, \tilde{T}) \underset{\text{(Lemma 1)}}{>} g^1(T_1, \tilde{T}) \underset{\text{(Def. 1)}}{=} V^1(T_1, \tilde{T})];$$

$$\forall T_1 \geqq \tilde{T},\ T_1 \neq \hat{\hat{T}}[V^1(\hat{\hat{T}}, \tilde{T}) \underset{\text{(Def. 1)}}{=} g^2(\hat{\hat{T}}, \tilde{T}) \underset{\text{(Lemma 1)}}{>} g^2(T_1, \tilde{T}) \underset{\text{(Def. 1)}}{=} V^1(T_1, \tilde{T})].$$

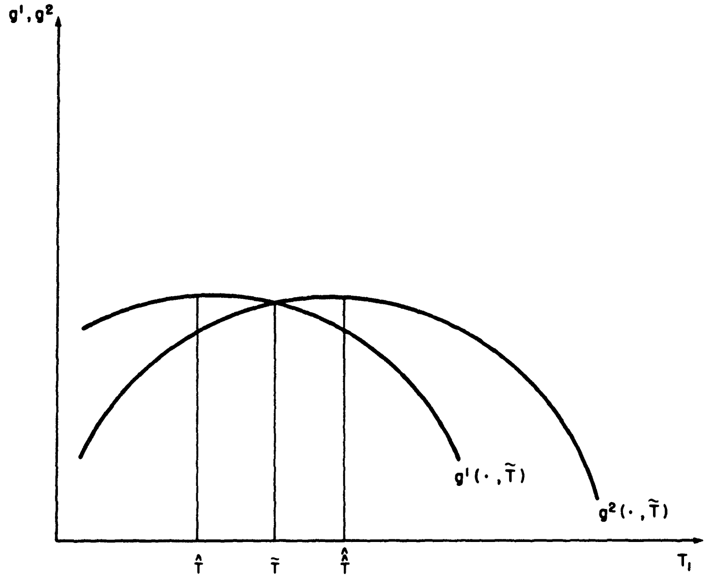

FIGURE 3

But since $g^1(\hat{T}, \tilde{T}) = g^2(\hat{\hat{T}}, \tilde{T})$ by Lemma 6, $V^1(\hat{T}, \tilde{T}) = V^1(\hat{\hat{T}}, \tilde{T}) > V^1(T_1, \tilde{T})$ $\forall T_1 \notin \{\hat{T}, \hat{\hat{T}}\}$. Thus $\phi_1(\tilde{T}) = \{\hat{T}, \hat{\hat{T}}\}$.

Case 3. $T_2 > \tilde{T}$.

$$\forall T_1 \leqq T_2, T_1 \neq \hat{T}[V^1(\hat{T}, T_2) \underset{\text{(Def. 1)}}{=} g^1(\hat{T}, T_2) \underset{\text{(Lemma 1)}}{>} g^1(T_1, T_2) \underset{\text{(Def. 1)}}{=} V^1(T_1, T_2)].$$

$$\forall T_1 \geqq T_2[V^1(\hat{T}, T_2) \underset{\text{(Def. 1)}}{=} g^1(\hat{T}, T_2) \underset{\text{(Lemma 6)}}{>} g^2(\hat{\hat{T}}, T_2) \underset{\text{(Lemma 1)}}{\geqq} g^2(T_1, T_2) \underset{\text{(Def. 1)}}{=} V^1(T_1, T_2)];$$

Thus $\forall T_1 \neq \hat{T}[V^1(\hat{T}, T_2) > V^1(T_1, T_2)]$. Hence $\phi_1(T_2) = \hat{T}$ for all $T_2 > \tilde{T}$. ||

The best response correspondence $\phi_1(T_2)$ is shown in Figure 4.

Proof of Theorem 1(a). By symmetry,

$$\phi_2(T_1) = \begin{cases} \hat{\hat{T}} & \text{for } T_1 < \tilde{T} \\ \{\hat{T}, \hat{\hat{T}}\} & \text{for } T_1 = \tilde{T}. \\ \hat{T} & \text{for } T_1 > \tilde{T} \end{cases}$$

It is apparent that the best response correspondences intersect at two distinct points, $(\hat{T}, \hat{\hat{T}})$ and $(\hat{\hat{T}}, \hat{T})$, as shown in Figure 5. ||

Recall that interior $\hat{T}$ and $\hat{\hat{T}}$ are determined by equations (3) and (4), repeated here for convenience:

$$g_1^1 = -(\pi_1 - \pi_0)\, e^{-r\hat{T}} - p'(\hat{T}) = 0 \tag{5}$$

$$g_1^2 = -(\pi_3 - \pi_2)\, e^{-r\hat{\hat{T}}} - p'(\hat{\hat{T}}) = 0. \tag{6}$$

Notice that the adoption dates depend on the profit rates only through the relevant opportunity costs of delaying adoption one period—$(\pi_1 - \pi_0)$ for $\hat{T}$ and $(\pi_3 - \pi_2)$ for $\hat{\hat{T}}$.

Comparative statics results are summarized in a corollary to Theorem 1.

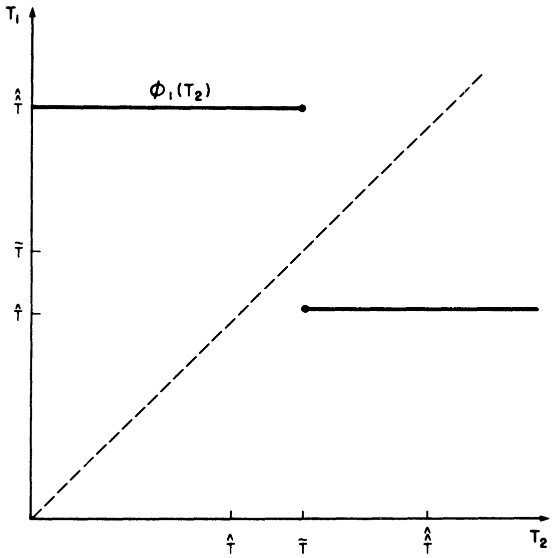

FIGURE 4

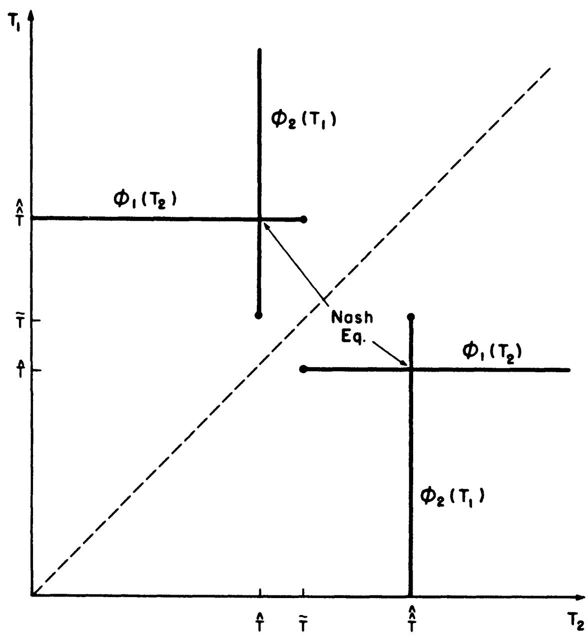

FIGURE 5

Corollary 1.
(a) $\partial \hat{T}/\partial(\pi_1-\pi_0)=e^{-r\hat{T}}/g_{11}^1<0$; *and*
(b) $\partial \hat{\hat{T}}/\partial(\pi_3-\pi_2)=e^{-r\hat{\hat{T}}}/g_{11}^2<0$.

That is, as the opportunity costs of waiting (foregone revenues) increase, adoption occurs earlier.

4. GENERALIZATIONS

This model may be generalized in at least three directions. First, one may extend the analysis to n identical firms. It is conjectured that this will result in $n!$ Nash equilibria obtained from each other by permuting the indices of the players. While rigorous analysis of the n-player game would probably not be difficult, it would be extremely messy and is not likely to add substantially to the implications of the 2-player case. It would be interesting, however, to examine comparative statics in n.

One can extend the analysis to include uncertainty regarding π_1, π_2 and π_3 simply by interpreting these as mean post-adoption profit opportunities. Finally, one can generalize the model to allow for asymmetric firms. Define π_0^i, π_i^i and π_3^i in the obvious way; also define π_j^i $(j \neq i)$ to be i's profit rate when j has adopted but i has not. Add a subscript to $p_i(T_i)$, the discounted price (plus adjustment costs) to indicate possible scale advantages. One need not indicate which firm is the more efficient—either at production or at technology implementation—at this point. However, the appropriate analogues of Assumptions 1–4 and Assumption 5a will be retained.

The payoff to player i is now

$$V^i(T_1, T_2) = \begin{cases} g^{i1}(T_1, T_2) & T_i \leqq T_j \\ g^{i2}(T_1, T_2) & T_i \geqq T_j \end{cases}$$

where

$$g^{i1}(T_1, T_2) = \int_0^{T_i} e^{-rt}\pi_0^i\,dt + \int_{T_i}^{T_j} e^{-rt}\pi_i^i\,dt + \int_{T_j}^{\infty} e^{-rt}\pi_3^i\,dt - p_i(T_i)$$

and

$$g^{i2}(T_1, T_2) = \int_0^{T_j} e^{-rt}\pi_0^i\,dt + \int_{T_j}^{T_i} e^{-rt}\pi_j^i\,dt + \int_{T_i}^{\infty} e^{-rt}\pi_3^i\,dt - p_i(T_i).$$

Lemmas 1–7 in no way depended on the symmetry of the players. Thus the best response correspondences become

$$\phi_i(T_j) = \begin{cases} \hat{\hat{T}}_i & T_j < \tilde{T}_i \\ \{\hat{\hat{T}}_i, \hat{T}_i\} & T_j = \tilde{T}_i \\ \hat{T}_i & T_j > \tilde{T}_i \end{cases}$$

where $\hat{T}_i$ and $\hat{\hat{T}}_i (> \hat{T}_i)$ maximize g^{i1} and g^{i2}, respectively, independent of T_j $(j \neq i)$; and $\tilde{T}_i \in (\hat{T}_i, \hat{\hat{T}}_i)$ is the unique solution of $g^{i1}(\hat{T}_i, t) = g^{i2}(\hat{\hat{T}}_i, t)$.

Now although it is easily shown that $\hat{\hat{T}}_i > \hat{T}_i$, $i = 1, 2$, one has no idea of the rankings of j's optimal dates relative to i's, especially since I have not specified which firm is "more efficient," nor even any notion of efficiency. However, one can still prove the following theorem characterizing Nash equilibrium with non-identical firms.

Theorem 2. *Under the appropriate analogues of Assumptions* 1–5*a*,

(a) *There is at least* 1 *(and no more than* 2*) Nash equilibrium.*

(b) *All Nash equilibria are asymmetric in the sense that firms will never adopt the new technology simultaneously.*

Proof.

(a) $\phi_1(T_2)$ is non-increasing and, when graphed in (T_1, T_2) space, consists of two horizontal half-lines which cover $[0, \infty)$; $\phi_2(T_1)$ is non-increasing and, when graphed in (T_1, T_2) space, consists of two vertical half-lines which cover $[0, \infty)$. They must intersect at least once and can intersect no more than twice.

(b) It is sufficient to show that matching is never a best response: $\forall T_j,\ T_j \notin \phi_i(T_j)$. Now $\forall T_j < \tilde{T}_i,\ \phi_i(T_j) = \hat{\hat{T}}_i > \tilde{T}_i$; for $T_j = \tilde{T}_i,\ \phi_i(\tilde{T}_i) = \{\hat{\hat{T}}_i, \hat{T}_i\}$ where $\hat{T}_i < \tilde{T}_i$ and $\hat{\hat{T}}_i > \tilde{T}_i$; and $\forall T_j > \tilde{T}_i,\ \phi_i(T_j) = \hat{T}_i < \tilde{T}_i$. Thus $\forall T_j,\ T_j \notin \phi_i(T_j)$. ||

Let us now relate this to Flaherty (1980). Using a slightly different model, Flaherty focuses on the question: will a larger (lower cost) firm commercialize a new technology earlier than a smaller (higher cost) firm? If the larger firm has a sufficiently large comparative advantage in implementing the new technology, then there exists a Nash equilibrium in which it will adopt sooner than a small firm. If the large firm has no comparative advantage, then there exists an equilibrium in which the smaller firm adopts first. However, in either case there may exist a second equilibrium in which the pattern is reversed. Analysis of the present model with asymmetric payoffs yields similar results (Reinganum, 1980*c*).

5. CONCLUSIONS

The choice of the date of adoption of a new technology has been modeled as a 2-person nonzero-sum game. Under Nash behavior we find that there exists a pair of equilibria in pure strategies (for identical firms). Each Nash equilibrium involves one firm adopting relatively early with the other adopting at a relatively late date, despite the facts that information is perfect and players are identical. For non-identical firms, a Nash equilibrium always exists and is asymmetric so long as the net value of being first is positive.

It is clear that a substantial modeling job remains—that is, to integrate optimal behavior (or strategic behavior) on the part of a seller of the new technology (see Stokey, 1979). Furthermore, there may be information regarding the value of the innovation to be gained by waiting. For an analysis of a single firm's decision problem in the face of such uncertainty, see Jensen (1979).

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NOTES

1. In light of the results on repeated games, I should specify that these are to be regarded as stage game, rather than supergame, equilibrium profits.

2. It is easy to verify that Assumptions 1–3 hold for simple demand curves (e.g. $P = a - bQ$, $P = a + b/Q$, $P = a - b \ln Q$) when marginal costs are constant. Whether Assumptions 1–3 hold for general cost and demand functions, and for a more general specification of cost reduction, is an open question at this time.

3. If $\lim_{t\to\infty} p'(t) \leqq 0$ and strict concavity is maintained, then $\forall T_2[\lim_{t\to\infty} g_1^1(t, T_2) \geqq 0]$ so $\forall T_2[g_1^1(t, T_2) > 0\ \forall t < \infty]$ and $\exists$ a maximizing value $\hat{T}$. Similarly, $\forall T_2[\lim_{t\to\infty} g_1^2(t, T_2) \geqq 0]$ so $\forall T_2[g_1^2(t, T_2) > 0\ \forall t < \infty]$ and $\exists$ a maximizing value $\hat{\hat{T}}$. If strict concavity is relaxed, optimal adoption dates $\hat{T}$ and $\hat{\hat{T}}$ may exist regardless of the sign of $\lim_{t\to\infty} p'(t)$. However, the existence of such dates would be difficult to verify without specifying the functional form of $p(t)$.

4. The notation $\forall T_i[\cdot]$ means "for all values of T_i in $[0, \infty)$, the bracketed statement is true".

5. It is shown in Reinganum (1980*c*) that there also exist symmetric Nash equilibria in both discrete and continuous mixed strategies. A Nash equilibrium in discrete mixed strategies is to play $\hat{T}$ with probability λ^* and $\hat{\hat{T}}$ with the complementary probability, where

$$\lambda^* = (\pi_1 - \pi_0)/\alpha - r[p(\hat{T}) - p(\hat{\hat{T}})]/\alpha[e^{-r\hat{T}} - e^{-r\hat{\hat{T}}}].$$

A Nash equilibrium in continuous mixed strategies is

$$F_i^N(t) = \Pr\{T_i \leqq t\} = (1/\alpha)[p'(t)\, e^{rt} + \pi_1 - \pi_0],$$

for $t \in [\hat{T}, \hat{\hat{T}}]$.

REFERENCES

FLAHERTY, M. T. (1980) "Timing Patterns of New Technology Adoption in Duopolistic Industries" (Manuscript).

JENSEN, R. A. (1979), "On the Adoption and Diffusion of Innovations Under Uncertainty" (Discussion Paper No. 410, Northwestern University).

KAMIEN, M. I. and SCHWARTZ, N. L. (1972), "Some Economic Consequences of Anticipating Technical Advance", *Western Economic Journal*, **10**, 123–138.

REINGANUM, J. F. (1980*a*), "Dynamic Games of Innovation", *Journal of Economic Theory* (forthcoming).

REINGANUM, J. F. (1980*b*), "A Dynamic Game of R & D: Patent Protection and Competitive Behavior", *Econometrica* (forthcoming).

REINGANUM, J. F. (1980*c*), "On the Diffusion of New Technology: A Game Theoretic Approach" (Social Science Working Paper No. 312 Pasadena, California Institute of Technology).

STOKEY, N. L. (1979), "Intertemporal Price Discrimination", *Quarterly Journal of Economics*, **93**, 355–371.