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Economic Instruction

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MICHAEL WATTS, Section Editor

A Geometric Solution of a Cournot Oligopoly with Nonidentical Firms

Jyotirmoy Sarkar, Barnali Gupta, and Debashis Pal

The first and still one of the most widely cited models of noncooperative oligopoly behavior is the Cournot model, developed by the French mathematician Augustin Cournot in 1838. The Cournot model is the fundamental model used to study strategic interactions among quantity-setting firms in an imperfectly competitive market. In the last two decades, there has been an explosion of Cournot-based models of strategic behavior to analyze various real-world phenomena ranging from horizontal mergers to intra-industry trade. A proper understanding of the Cournot model of imperfect competition and strategic interactions among firms in various contexts is thus essential.

The Cournot model is an integral part of undergraduate microeconomics and industrial organization textbooks. The typical approach is to first use market demand and cost functions to derive the "reaction functions" or the "best-response functions" of two competing firms. Next, these reaction functions are drawn together on the same graph to determine their intersection point, which corresponds to the equilibrium quantities. Finally, the equilibrium quantities are substituted in the demand function to determine the equilibrium price and the equilibrium profits of the firms (see, Carlton and Perloff 1994; Eaton and Eaton 1995; Katz and Rosen 1994; Mansfield 1994; Martin 1993; Pindyck and Rubinfeld 1995; Salvatore 1997; Schotter 1997; Varian 1996).

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This method of using reaction functions to solve a Cournot model does an excellent job of demonstrating the solution technique. Some drawbacks are inherent in this approach, however. It does not allow the graphical solution of a Cournot model with more than two firms, because such a solution would require a graph with more than two dimensions. To solve a Cournot model with more than two firms, textbook authors rely on algebra. Because the algebra becomes quite complicated with nonidentical firms, they typically do not present the solution for a Cournot oligopoly with more than two nonidentical firms. Because undergraduate students often prefer graphical methods to algebra and calculus, the task is to develop a simple graphical technique to solve a Cournot oligopoly with any number of nonidentical firms.

Fulton (1996) developed graphical methods to solve a Cournot oligopoly with multiple firms. He presented elegant ways to graphically derive the reaction functions of the firms, and then the reaction functions were used to determine the equilibrium price and quantities. Using the reaction function approach, he also presented graphical methods to solve a Stackelberg model with multiple firms. Fulton's method, however, relies on drawing the reaction functions and therefore cannot be used to solve a Cournot model with more than two nonidentical firms.

We present a simple graphical technique to solve a Cournot oligopoly for any number of nonidentical firms. The technique is different from that in Fulton and avoids the necessity of drawing reaction functions. The method draws heavily on high school level geometry and requires no higher-level mathematics. Furthermore, it provides an easy way of visualizing the impact of changing various external parameters on equilibrium price and quantities. The graphical technique is complementary to the standard solution technique and will be a useful teaching tool in both intermediate microeconomics and industrial organization courses.

THE MODEL

Consider a market with N firms that produce a homogeneous product. Let q_i be the output produced by firm i and

$$Q = \sum_{i=1}^{N} q_i$$

be the total output produced in the market. Let $P(Q) = a - bQ$ be the inverse demand function, $a > 0$ and $b > 0$.[1] We assume that firm i ($i = 1, \ldots, N$) has a constant marginal cost of production, denoted by c_i.[2] Firms simultaneously decide how much to produce, and the market-clearing price is determined from the inverse demand function. Firm i ($i = 1, \ldots, N$) chooses q_i to maximize its profit, $\Pi_i = q_i[P(Q) - c_i]$.

SOLUTION

First, we provide a graphical method of determining the equilibrium price and equilibrium (total) quantity. Once the equilibrium price is known, we determine the level of output produced by each firm.

Determination of Equilibrium (Total) Quantity and Equilibrium Price

Note that firm i's first-order condition of profit maximization is

$$P(Q) - c_i - bq_i = 0. \tag{1}$$

Adding the first-order conditions for all firms yields

$$NP(Q) - bQ = \sum_{i=1}^{N} c_i.$$

Dividing throughout by N yields $P(Q) - bQ/N = \bar{c}$, where

$$\bar{c} = (\sum_{i=1}^{N} c_i) / N.$$

Substituting $P(Q) = a - bQ$ in the above expression and simplifying yields

$$a - [(N + 1)/N]\, bQ = \bar{c}. \tag{2}$$

From equation (2) it follows that the equilibrium (total) quantity Q^* can be found by equating the average of the marginal costs in the industry, $\bar{c}$, with a line that has the same intercept as the demand curve and a slope $(N + 1)/N$ times that of the demand curve; that is, with slope $-[(N + 1)/N]b$. This line can easily be determined by extending the demand curve down to the horizontal axis and then dividing the distance between the origin and the point where the demand curve cuts the horizontal axis into $(N + 1)$ equal segments. Drawing a line from a to the end of the Nth segment (moving right from the origin) gives a line with a slope of $(N + 1)/N$ times that of the demand curve.

In Figure 1, B is the point of intersection between the straight line from a with slope $-[(N + 1)/N]b$ and the horizontal straight line from $\bar{c}$. A vertical straight line is drawn through B. This vertical line cuts the horizontal axis at Q^*, the equilibrium (total) quantity. The corresponding equilibrium price is P^*.

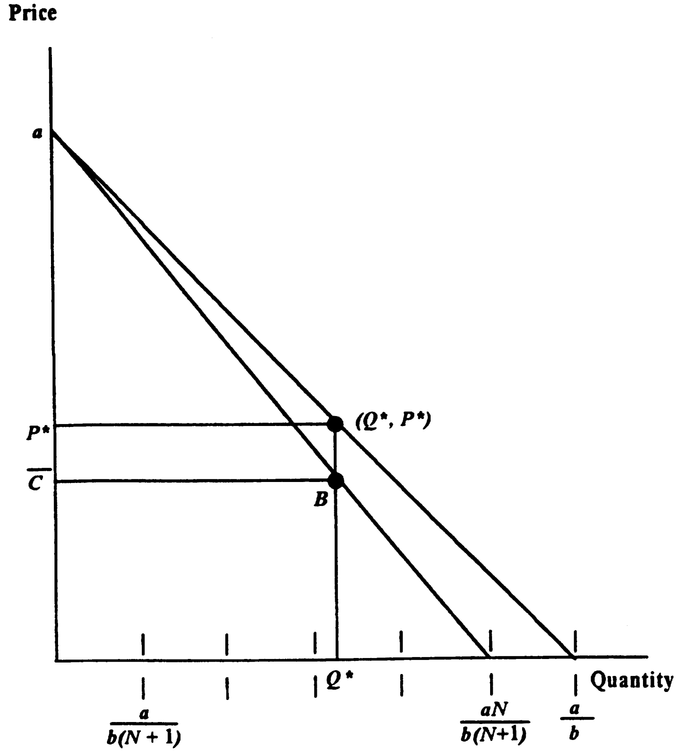
It is interesting that, if there is only one firm ($N = 1$) with marginal cost c, then equation (2) reduces to $a - 2bQ = c$, the standard rule that marginal revenue equals marginal cost for profit maximization of a monopolist. Also, when $N = 1$, Figure 1 reduces to the familiar diagram used to determine the profit-maximizing output and price of a monopolist.

It is easy to see that the output under a Cournot oligopoly lies between the competitive output and the monopoly output, and the Cournot output converges to the competitive output as N becomes larger. To illustrate, we assume $c_i = c$ for all firms. In Figure 2, Q^* is the Cournot output, Q^M is the monopoly output, and Q^C is the competitive output. Clearly, Q^* lies between Q^M and Q^C. Also, as N becomes larger, $(N + 1)/N$ converges to 1, and the straight line from a with slope $[(N + 1)/N]b$ rotates toward the demand curve. As a result, as N becomes larger, Q^* converges to Q^C.

Determination of Individual Firm Output

From equation (1), the first-order conditions for profit maximization imply that $q_i^* = (P^* - c_i)/b$ for all $i = 1, \ldots, N$. In turn, this implies that $P^* - bq_i^* = c_i$. Thus, a line through P^* parallel to the demand curve (that is, with slope $-b$) inter-

FIGURE 1
Determination of Equilibrium (Total) Quantity and Price



sects the horizontal straight line through firm i's marginal cost at point (q_i^*, c_i). A vertical straight line drawn from this point cuts the horizontal axis at q_i^*.

Figure 3 illustrates output determination for individual firms when $N = 3$. The line P^* D is parallel to the demand curve. It cuts the horizontal line through firm i's ($i = 1, 2, 3$) marginal cost at B_i. A vertical line drawn through B_i intersects the horizontal axis at q_i^*, firm i's equilibrium output. Let E_i be the point where the vertical line through B_i intersects the horizontal line through P^*. The area of the rectangle $P^*E_iB_ic_i$ equals $q_i(P^* - c_i)$, the profit of firm i, because $(P^* - c_i)$ is firm i's profit per unit of output and q_i is its amount of output.

DISCUSSION

Several results from the Cournot model are illustrated in Figure 3. Some of these results are cumbersome to show with algebra or calculus. In this section, we demonstrate a few. First note from Figure 3 that firms with higher marginal costs produce less. Also, $(q_i^* - q_k^*)$ is proportional to $(c_k - c_i)$. That is, the difference in outputs between firms is proportional to the difference between their

FIGURE 2
Monopoly, Cournot, and Competitive Output and Prices

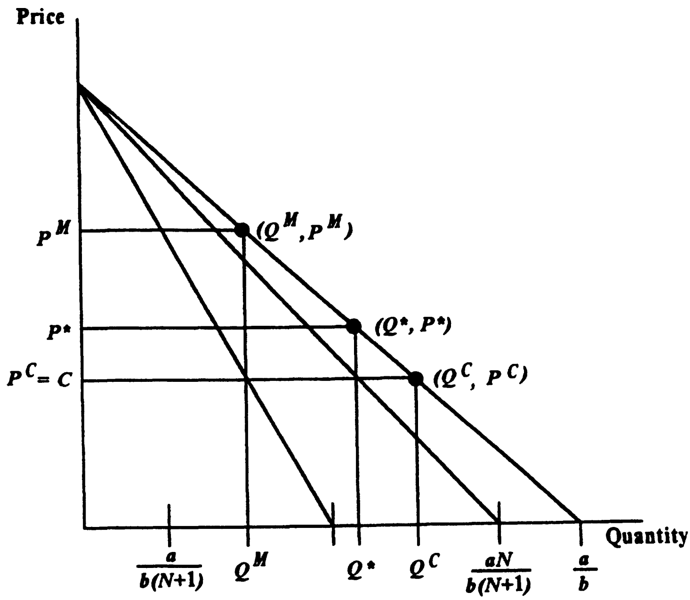
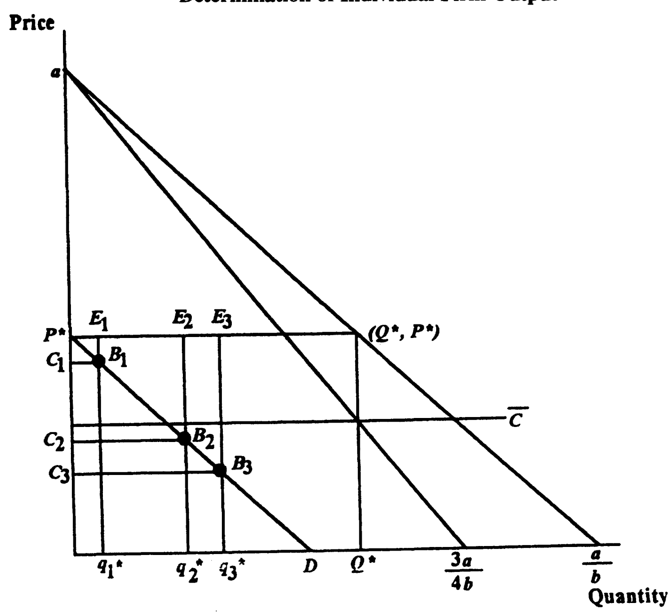


FIGURE 3
Determination of Individual Firm Output



marginal costs. This result follows from Figure 3 because for any $i \neq k$, $(c_k - c_i)$ equals the height, and $(q_i^* - q_k^*)$ equals the base of a right-angled triangle whose hypotenuse B_iB_k has a constant slope of $-b$. Similarly, the equilibrium output of firm i, q_i^* is proportional to $(P^* - c_i)$, the profit per unit of output for firm i. These results, also follow directly from the first-order condition $q_i^* = (P^* - c_i)/b$.

Second, consider a redistribution of marginal costs among the firms. That is, c_i may increase and c_k may decrease, but

$$\sum_{i=1}^{N} c_i$$

and hence $\bar{c}$ remain the same. From Figure 1, note that $Q^*$and P^* do not change. A redistribution of marginal costs leading to a change in asymmetry among the firms, however, has an interesting effect on the aggregate profits. Consider three firms with $c_1 > c_2 > c_3$, as in Figure 3. Suppose c_1 goes up by the small positive amount ε, and c_3 goes down by ε. It follows that the output of firm 1 goes down and the output of firm 3 goes up, but the total output, the market price, and the output of firm 2 do not change. Therefore, the decrease in the output of firm 1 equals the increase in the output of firm 3. Because the profit per unit of output is more for firm 3, the increase in profit for firm 3 is more than the decrease in profit for firm 1. As a result, the aggregate industry profit goes up. This can be seen in Figure 3 by comparing the changes in the respective areas of $P^*E_1B_1c_1$ and $P^*E_3B_3c_3$. Intuitively, an increase in asymmetry increases the market shares in favor of the more efficient firms and thus increases the aggregate industry profit.

A third insight that can be derived from the graphical approach is the impact of an industry-wide change in the marginal cost of production. Suppose the marginal cost of each firm increases by the same amount ε. This can happen if, for instance, the cost of an input increases or an excise tax is imposed on the producers. As a result of this increase in marginal costs, $\bar{c}$ increases by ε and from Figure 1 it can be checked that the equilibrium price increases by $N\varepsilon/(N + 1)$ and the equilibrium total quantity decreases by $N\varepsilon/[(N + 1)b]$. Once the new equilibrium price is determined, the individual equilibrium quantities may be obtained as depicted in Figure 3, replacing c_i by $(c_i + \varepsilon)$ and P^* by $P^* + N\varepsilon/(N + 1)$. That is, the new equilibrium quantity for firm i, $\hat{q}_i$, is the solution to the equation

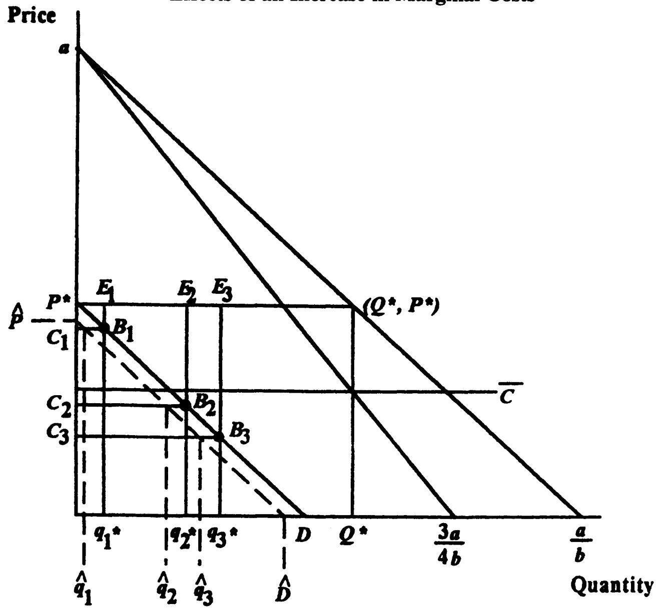
$$P^* + N\varepsilon/(N + 1) - b\hat{q}_i = c_i + \varepsilon. \tag{3}$$

However, there is a simpler way to obtain the individual equilibrium quantities. Note that equation (3) can be rewritten as $P^* - [\varepsilon/(N + 1)] - b\hat{q}_i = c_i$. Thus, a line through $P^* - \varepsilon/(N + 1)$ parallel to the demand curve (that is, with slope $-b$) intersects the horizontal straight line through firm i's marginal cost at point $(\hat{q}_i, c_i)$. A vertical straight line drawn from this point cuts the horizontal axis at $\hat{q}_i$.

In Figure 4, $\hat{P}\hat{D}$ is a line parallel to the demand curve where $\hat{P} = P^* - \varepsilon/(N + 1)$. Because $\hat{P}$ is $\varepsilon/(N + 1)$ units below P^*, every firm's profit per unit output decreases by $\varepsilon/(N + 1)$. A lower per unit profit together with lower quantity decreases the total profit of each firm, and thus decreases the total industry profit.

More interesting, because $\hat{P}\hat{D}$ is parallel to P^*D, it follows from Figure 4 that

FIGURE 4
Effects of an Increase in Marginal Costs



irrespective of the initial equilibrium output, every firm's equilibrium output decreases by the constant amount $\varepsilon/[(N + 1)b]$. Because firm 3 has a higher equilibrium output before the cost increase, an industry-wide cost increase expands its market share. This result confirms that in a Cournot model, an industry-wide cost increase expands the market shares of the more efficient firms.

CONCLUSION

We have presented a simple graphical technique to solve a Cournot oligopoly with any number of nonidentical firms. Because the usual reaction-function approach to solve a Cournot model graphically cannot be used for more than two firms and the algebraic approach becomes mathematically complicated for nonidentical firms, this method can be a very useful learning tool for students and a helpful teaching tool for instructors. Moreover, it can be used to illustrate several interesting properties of a Cournot model, which are otherwise mathematically tedious to derive. Finally, the method presented here requires no more than high school geometry and can easily be used in both intermediate microeconomics and industrial organization courses.

For simplicity, we have presented the technique for a linear market demand. In the appendix, we demonstrate how it can also be used for nonlinear demands. Also, we assumed constant marginal costs of production. The method, however, is valid for some forms of increasing marginal costs. For example, it can be modified to include parallel, linear increasing marginal costs of production.

Further research with this approach would certainly be useful. It would be interesting to see what other properties of a Cournot model can be studied graphically and whether this technique can be extended to solve other oligopoly models such as the Stackelberg model. That would be an interesting exercise for students of intermediate microeconomics and industrial organization.

APPENDIX
COURNOT EQUILIBRIUM FOR NONLINEAR DEMAND

Let $P(Q)$ be the inverse demand function. We assume that

$$P'(Q) < 0 \text{ and } 2P'(Q) + QP''(Q) < 0. \tag{A1}$$

Note that firm i ($i = 1, \ldots, N$) chooses q_i to maximize its profit $\Pi_i = q_i[P(Q) - c_i]$. Therefore, the first-order condition for profit maximization is $P(Q) + q_iP'(Q) = c_i$. Adding the first-order conditions for all firms, we get

$$NP(Q) + QP'(Q) = \sum_{i=1}^{N} c_i. \tag{A2}$$

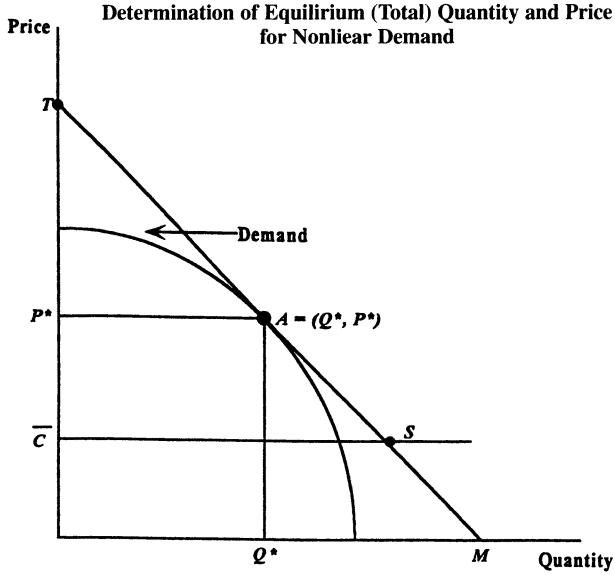
Note that, by assumption equation (A1), the left-hand side of equation (A2) is monotonically decreasing in Q. Thus, if

$$NP(0) > \sum_{i=1}^{N} c_i,$$

equation (A2) has a unique solution Q*, which is the equilibrium (total) quantity. The corresponding equilibrium price can be determined from the demand curve.

Figure (A1) illustrates how the equilibrium (total) quantity Q^* can be determined graphically. Find the point A on the demand curve such that the tangent TM, to the demand curve

FIGURE A1
Determination of Equilirium (Total) Quantity and Price for Nonliear Demand



at A, intersects the vertical axis at T and the horizontal line through $\bar{c}$ at S and $TA/AS = N$. Therefore, point A corresponds to the equilibrium quantity and price (Q^*, P^*).

Once the equilibrium price is known, the level of output produced by each firm can be determined by treating the straight line TM as the demand curve and following the technique already developed for linear demand. That is, a line through P^* parallel to the straight line TM intersects the horizontal straight line through firm i's marginal cost at point (q_i^*, c_i). A vertical straight line drawn from this point cuts the horizontal axis at q_i^*.

NOTES

1. Our solution technique is valid for nonlinear demand as well (see the Appendix).
2. Our technique is also valid when the firms have parallel, linear increasing marginal cost of production. That is, $c_i(q_i) = \gamma_i + \delta_i q_i + \mu q_i^2$. We would be happy to provide a proof.

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