Urban Land Prices under Uncertainty

By SHERIDAN TITMAN*

Land prices in west Los Angeles are among the highest in the United States. Yet, we can observe a number of vacant lots and grossly underutilized land in this area. A good example of this is a parking lot, owned by the University of California-Los Angeles, in an area of Westwood where land has been known to sell for more than $100 per square foot. The university could probably raise a considerable amount of money by selling two-thirds of the parking lot and constructing a parking structure on the remaining land to satisfy the demand for parking. Although this may be one of the best examples of underutilized land in west Los Angeles, it is by no means the only example. There are many underutilized and vacant urban lots throughout Los Angeles and the rest of the world, held by private investors who presumably wish to maximize their wealth.

The fact that investors choose to keep valuable land vacant or underutilized for prolonged periods of time suggests that the land is more valuable as a potential site for development in the future than it is as an actual site for constructing any particular building at the present time. Hence, in order to understand why certain urban lots remain vacant, we must determine how the land is valued under the two alternatives. Valuing the land as a site for constructing a particular building at the current time is fairly straightforward. It is simply the market value of the building (including the land) minus the lot preparation and construction costs (this is referred to in the real estate literature as residual value). However, valuing the vacant land as a potential building site is not as straightforward since the type of building that will eventually be built on the land, as well as the future real estate prices, are uncertain.

The model developed in this paper provides a valuation equation for pricing vacant lots of this type. Although the model is very simple, it provides strong intuition about the conditions under which it is rational to postpone building until a future date. Furthermore, the pricing model can be adapted to provide realistic estimates of urban land values in much more complex settings.

The notion that it is often optimal to delay irreversible investment decisions has previously been considered in the environmental economics and capital investments literature.[1] The basic intuition in these papers is that it may be advantageous to wait for additional information before deciding upon the exact specifications of the investment project. While the authors demonstrate that it is often valuable to delay investment, and maintain the option to choose a better investment project in the future, they do not explicitly show how this option affects the value of other related assets in their models.

This paper adapts the methods first used by Fisher Black and Myron Scholes (1973) and Robert Merton (1973), to value options and other derivative securities, to determine explicit values for vacant urban land. The valuation model is particularly close in its approach to the binomial option pricing models of John Cox, Stephen Ross, and Mark Rubinstein (1979), and Richard Rendleman and Brit Bartter (1979). The intuition being that a vacant lot can be viewed as an option to purchase one of a number of different possible buildings at exercise prices that are equal to their respective construction costs.

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[1] See, for example, John Krutilla (1967), Alex Cukierman (1980), Douglas Greenley, Richard Walsh, and Robert Young, (1981), and Ben Bernanke (1983).

This approach provides a valuation formula that is a function of observable variables and is independent of the investor's preferences.

This valuation technique should be contrasted to the standard textbook approach to valuing vacant land under uncertainty.[2] Richard Ratcliff (1972), for instance, suggests that appraisers determine the most probable future use of the land, appraise the property as of that future time and that use, and then discount this future value to the present. This method ignores the fact that the type of building that will be constructed in the future is generally unknown, and will be determined by real estate prices at that time. The analysis in this paper demonstrates that the amount of uncertainty about the type of building that will be optimal in the future is an important determinant of the value of vacant land. If there is a lot of uncertainty about future real estate prices, then the option to select the type of building in the future is very valuable. This makes the vacant land relatively more valuable and makes the decision to develop the land at the current time relatively less attractive. However, if there is very little uncertainty about future real estate prices, the option to select the appropriate type of building in the future is relatively less valuable. In this case, the decision to develop the land at the current time is relatively more attractive.

My analysis provides more than just a novel method for valuing land under uncertainty. It enables us to address issues, previously unexplored, that pertain to the effect of uncertainty on real estate markets. My results relating to how uncertainty about future real estate prices affect current real estate activities has important policy implications. For example, the analysis suggests that government action intended to stimulate construction activities may actually lead to a decrease in such activities if the extent and duration of the activity is uncertain. The analysis also has policy implications regarding the imposition of height restrictions on buildings. It is shown that the initiation of height restrictions, perhaps for the purpose of limiting growth in an area, may lead to an increase in building activity in the area because of the consequent decrease in uncertainty regarding the optimal height of the buildings, and thus has the immediate affect of increasing the number of building units in an area.

The paper is organized as follows: Section I examines the type of building, characterized by its size, that will be built at a given date if the land is to be developed at that time. Section II presents a simple two-date, two states of nature, model for determining the value of the vacant land for the case where the future price of building units, and hence the size of the building that is to be constructed, is uncertain. A simple numerical example that illustrates this valuation technique is presented in Section III. Section IV presents a comparative static analysis of this valuation model which includes, among other things, an analysis of the effect of uncertainty on vacant land value. Section V examines a model where the current price and rental rate on building units as well as land values are endogenous and Section VI provides a numerical example which illustrates how the valuation technique can be applied to value land with many possible building dates and many possible states of nature corresponding to each date.

I. The Optimal Building Size

Buildings, in this model, are characterized by their size, or number of units, q. The cost of constructing a building on a given piece of land, C, is an increasing and convex function of the number of units, that is, $dC/dq > 0$ and $d^2C/dq^2 > 0$. The rationale for the second assumption is that as the number of floors in a building increases, labor costs per floor increase and the foundation of the building must be stronger. It is also assumed that subsequent to completing a building of

[2] I am unaware of any extant land pricing models that consider uncertainty. However, Donald Shoup (1970), Chapman Findlay and Hugh Howson (1975), and James Markusen and David Scheffman (1978) have examined some of the issues analyzed here within certainty models. Also, René Stulz (1982) suggested that the model he developed for pricing options to purchase one of two risky assets could be applied to price land in some specific cases.

a certain size, it is prohibitively expensive to add additional building units.

Given these assumptions, the building size that maximizes the wealth of a landowner who wishes to construct a building at the present time will satisfy the following maximization problem:

$$\text{(1)} \qquad \underset{\{q\}}{\text{Max}}\, \Pi(p_0) = p_0 q - C(q),$$

where p_0 is the current market price per building unit.

Differentiating (1) with respect to q, it follows that the solution to this maximization problem is to choose a building size which satisfies the condition,

$$\text{(2)} \qquad dC/dq = p_0.$$

The building size that satisfies this equality will be denoted q^*. Given this optimal decision, it follows directly that the value of the land for building at the present time, $\pi(p_0)$, is an increasing and convex function of p_0. It should be noted that the convexity results because the landowner can change q^* in response to changes in p_0.

I will later demonstrate, within a more specialized model, that because of this convexity property, greater uncertainty about the future unit price of buildings increases the current value of vacant land. The basic intuition behind this result can be seen by comparing the expected value of the land for building at date 1, over uncertain realizations of $\tilde{p}_1$ with the value of the land given a known date 1 price of $\hat{p}_1 = E(\tilde{p}_1)$. It follows from Jensen's inequality that

$$\text{(3)} \qquad E(\Pi(\tilde{p}_1)) > \Pi(E(\tilde{p}_1)).$$

Hence, uncertainty increases the expected future value of the vacant land. This implies that uncertainty causes current vacant land values to increase at least for the case where investors are risk neutral.[3]

[3]For the special cases where $\tilde{p}_1$ is normally distributed, or where $C(q)$ is quadratic, the expected future value of land is monotonically increasing in the variance of $\tilde{p}_1$.

II. Valuing Urban Land under Uncertainty

Here I present a simple model for valuing land under uncertainty. Although the model makes no assumptions about investor preferences, other simplifying assumptions are made. The model consists of only two dates, so if the landowner chooses not to build at the present date (date 0), he or she will develop the land at date 1 if $\pi(p_1) > 0$. Holding vacant land is assumed to generate no revenues or costs. Uncertainty, in this model, enters in a very simplistic manner. First, the only source of uncertainty pertains to the market price of building units. Per unit construction costs are known and constant. Furthermore, $\tilde{p}_1$, the date 1 price of building units takes on one of only two possible values, p_h and p_l, where $p_h > p_l$. Given that building units can take on only two possible prices on the second date and building costs are constant, it follows that the land at date 1 can take on only two possible values, $\pi(p_h)$ and $\pi(p_l)$. It should be noted that these simplifying assumptions are relaxed considerably in Section VI. It is also assumed that a risk-free asset exists with a return of R_f. The per unit rental rate, R_t, is initially assumed to be exogenous; however, this variable is determined endogenously in the model presented in Section V. Finally, it is assumed that markets are perfect in that there are no taxes, no transaction costs, and no short-selling restrictions.[4]

The vacant land can be considered what the finance literature refers to as a contingent or derivative security. Its date 1 value is completely determined by (or derived from)

[4]The assumption of frictionless markets, generally assumed in models of security prices, is considered by some to be less realistic when applied to real estate markets. However, it should be noted that securities represent indirect claims on factories and equipment that are probably much less liquid than real estate. Yet we can price these assets as if they were perfectly liquid because the securities are traded on (almost) frictionless markets. Similarly, a large fraction of real estate is held by publicly traded firms. If the real estate investments of these firms are chosen in a manner consistent with value maximization, then real estate prices will be determined in equilibrium as if markets were really frictionless.

an exogenously priced asset, the date 1 price of building units. In the finance literature, options and other contingent securities are valued by forming a hedge portfolio, consisting of the risk-free asset and the exogenously priced primitive asset, that is perfectly correlated with the contingent security. In the absence of riskless arbitrage, the contingent security must have the same value as this hedge portfolio.

The vacant land can be similarly valued in this model. Since there exist three investments (land, building units, and the risk-free asset) that take on at most two possible values, the returns of the vacant land can be exactly duplicated by a linear combination of the returns of the building units and the risk-free asset. Hence, in the absence of riskless arbitrage, the price of the vacant land can be determined as a function of these investments.

An easy way to solve this pricing problem is to first determine the state prices, (i.e., the cost at date 0 of receiving one dollar in one of the two date 1 states of nature and zero dollars in the other), and then sum the products of these state prices and the land values in the two states of nature. These state prices, s_h and s_l, must satisfy the following two equations that express the date 0 price of building units and the price of a discount bond as functions of their date 1 cash flows:

$$(4) \quad p_0 = s_h p_h + s_l p_l + R_t(s_h + s_l)$$

$$(5) \quad 1/(1+R_f) = s_l + s_h.$$

Solving these equations yields the following state prices for high and low price states of nature, respectively:

$$(6) \qquad s_h = \frac{p_0 - (p_l + R_t/1 + R_f)}{p_h - p_l}$$

and

$$(7) \qquad s_l = \frac{(p_h + R_t/1 + R_f) - p_0}{p_h - p_l}.$$

Given these state prices, it follows that if no opportunities for riskless arbitrage exists, the price of vacant land at date 0 must be

$$(8) \qquad V = \Pi(p_h)s_h + \Pi(p_l)s_l.$$

If the value of the vacant land, as specified in equation (8), exceeds the profit from building at the present date, $\Pi(p_0)$, the wealth-maximizing landowner will choose to have the land remain vacant. Otherwise, he or she will build at date 0 the size building that satisfies equation (2).

III. A Simple Numerical Example

Consider the example where an investor owns a lot that is suitable for either six or nine condominium units. The per unit construction costs of the building with six and nine units is \$80,000 and \$90,000, respectively. The current market price of the units is \$100,000. The per year rental rate is \$8,000 per unit (net of expenses) and the risk-free rate of interest for the year is 12 percent. If market conditions are favorable next year, the condominiums will sell for \$120,000; if conditions are unfavorable, they will sell for only \$90,000.

Since the marginal cost, per unit, of building nine rather than six units is \$110,000, the investor will build a six-unit building and realize a profit of \$120,000 if he builds at the current time. However, if he chooses to wait one year to build, he will construct a six-unit building if market conditions are unfavorable and realize a total profit of \$60,000, and will build a nine-unit building and realize a total profit of \$270,000 if favorable market conditions prevail. Substituting these numbers into equation (8) yields a current value for this land, if it is to remain vacant until next year, of \$141,071. Since this is greater than the profit that would be realized by building immediately, it is better to keep the land vacant.

If the land sells for less than this amount, investors can earn arbitrage profits by purchasing the land, and hedging the risk by short-selling the condominium units. For example, if the land sold for \$120,000, investors could realize a risk-free gain with no initial investment by purchasing the land, short-selling seven condomium units, and investing the net proceeds from the transactions in the risk-free asset. The seven condominium units completely hedges the risk from owning the vacant land since the difference between the value of the units in the good and bad states of nature, \$210,000, exactly offsets the difference in land values in the two states. Hence, the above investment yields a risk-free gain of \$23,600. Since such gains cannot exist in equilibrium, investors will bid up the price of the land to its equilibrium value of \$141,071.

IV. Comparative Statics

The above numerical example illustrates the effects of the current price of the building units, the interest rate, and the rental rate on the current value of vacant land. Recall that in order to hedge the risk from owning the vacant land, individual building units were sold, with the proceeds invested in the risk-free asset. If the price of the building units increases, the proceeds from the short sale increase, so the vacant land becomes more valuable. Similarly, if the interest rate increases, the income from the risk-free asset increases so the vacant land becomes less valuable. Conversely, if the rental rate increases, the cost of maintaining the short position increases, so the value of the vacant land decreases.

These comparative static results can be shown formally by differentiating equation (8) under the assumption that p_h and p_l are fixed:

$$\text{(9a)}\quad \partial V/\partial p_0 = \frac{\Pi(p_h) - \Pi(p_l)}{p_h - p_l} > 0,$$

$$\text{(9b)}\quad \partial V/\partial R_f = \frac{\Pi(p_h)(p_l + R_t) - \Pi(p_l)(p_h + R_t)}{(p_h - p_l)(1 + R_f)^2} \gtrless 0,$$

$$\text{(9c)}\quad \partial V/\partial R_t = \frac{\Pi(p_l) - \Pi(p_h)}{(p_h - p_l)(1 + R_f)} < 0.$$

The preceding analysis implicitly assumes that the current price and rental rate on building units are unaffected by changes in the risk-free rate. Alternatively, we can examine the case where R_t is constrained to equal $R_f p_0$. A change in the risk-free rate accompanied by a proportional change in the rental rate can then be analyzed by substituting $R_f p_0$ for R_t in equation (8) to yield

$$\text{(8')}\quad V = \Pi(p_h)\left(\frac{p_0 - p_l}{(p_h - p_l)(1 + R_f)}\right) + \Pi(p_l)\left(\frac{p_h - p_0}{(p_h - p_l)(1 + R_f)}\right).$$

It is clear from the above equation that the value of the vacant land decreases if an increase in interest rates is accompanied by a corresponding increase in rental rates.

The valuation technique presented in Section IV above also enables us to analyze the effect of increased uncertainty on land values. This is done by considering the effect of increasing the spread between p_h and p_l in such a way that state prices remain constant, and are consistent with both current rental rates and the prices of building units remaining constant. Hence, the effect of uncertainty on land values established here is applicable to cross-sectional comparisons holding current building prices constant.

One can easily verify that if p_h increases by x dollars and p_l decreases by xs_h/s_l dollars, the state prices remain unchanged. Also, the value $p_h s_h + p_l s_l = p_0 - R_t/1 + R_f$ remains unchanged. This is consistent with, but does not require, p_0 and R_t to remain unchanged. However, the value of vacant land,

$$\text{(10)}\quad V = \Pi(p_h + x)s_h + \Pi(p_l - (xs_h/s_l))s_l,$$

is an increasing function of x. This can be seen by differentiating V, in equation (10), with respect to x:

$$dV/dx = \Pi'(p_h + x)s_h + \Pi'(p_l - (xs_h/s_l))s_h.$$

It follows from the convexity of $\Pi(p)$ that

$$dV/dx > 0 \quad \text{since}$$

$$\Pi'(p_h + x) > \Pi'(p_l - (xs_h/s_l)).$$

This result indicates that if the amount of uncertainty increases, the value of the vacant land increases, decreasing the relative attractiveness of constructing a building at the current time. Developing the land at the current time becomes less attractive because the increased uncertainty about future prices makes the size of the building that will be optimal at the future date more uncertain, which in turn makes it more likely that the optimal building size at the current time will be suboptimal in the future. If the building size (q^*) that will be constructed in the future is known, perhaps because of height restrictions, then the amount of uncertainty about future prices will not enter the decision as to whether to build now or to build in the future. The decision will instead be determined by a comparison between the rental rate and the return from investing the construction expenses in the risk-free asset. This can be seen by comparing the value of the land for constructing a building with q^* units at the current time period:

$$\Pi = p_0 q^* - C(q^*), \tag{11}$$

with its value as a building site for next period:

$$V = s_h[p_h q^* - C(q^*)] + s_l[p_l q^* - C(q^*)]. \tag{12}$$

Substituting equation (7) into (12) yields

$$V = p_0 q^* - R_t q^*(s_h + s_l) - C(q^*)(s_h + s_l), \tag{13}$$

which suggests that the building should be constructed at the present date if and only if,

$$(C(q^*) + R_t q^*)/(1 + R_f) > C(q^*),$$

which simplifies to

$$R_t q^* > R_f C(q^*). \tag{14}$$

Since condition (14) is less restrictive than the condition $\Pi(p_0) > V$ (for the case where there are no building restrictions), a particular piece of land may be developed at the present date (if height restrictions are imposed), in circumstances under which it would not be developed otherwise. Hence, the imposition of height restrictions can conceivably have the immediate effect of increasing the number of building units in a particular area.

The effects of changes in future building prices, which do lead to changes in current building prices, can also be examined within this model. An increase in p_h, holding p_l, s_l, s_h, and R_t constant, will increase p_0 by the amount s_h (see equation (4)), which in turn will increase the profit from developing the land at the current date by the amount

$$d\Pi/dp_h = \Pi'(p_0)s_h. \tag{15}$$

From equation (8), this increase in p_h leads to an increase in the value of the vacant land of

$$dV/dp_h = \Pi'(p_h)s_h. \tag{16}$$

If p_h exceeds p_0, $\Pi'(p_h)$ will exceed $\Pi'(p_0)$ since $\Pi(\cdot)$ is convex. In this case, an increase in building prices in the good state of nature increases the current value of the vacant land relative to its value if developed. Hence, it becomes less attractive to build at the current date. In the less likely case where the price of building units in the favorable state of nature is lower than the current price, an increase in p_h makes it more attractive to build at the current date.

Similarly, a decrease in p_l, holding the other variables constant, decreases current building unit prices by s_l, which in turn leads to a decrease in the profit from developing the land at the current date by the amount

$$d\Pi/dp_l = \Pi'(p_0)s_l. \tag{17}$$

This decrease in p_l leads to a corresponding decrease in the vacant land value of

$$dV/dp_l = \Pi'(p_l)s_l. \tag{18}$$

It follows, from the above equations, that a decrease in p_l will lead to a decrease in the profit from developing the land at the current date that is greater (less) than the corresponding decrease in the value of the vacant land if p_0 exceeds (is less than) p_l. The above analysis suggests that any increase in the $p_h - p_l$ spread makes it relatively more valuable to delay developing the land as long as $p_h > p_0 > p_l$. This conforms to the basic intuition that increased uncertainty increases the value of having open alternatives. However, this intuition does not necessarily hold when either $p_0 > p_h$, or $p_0 < p_l$.

V. A Simple Examination

Here I present a simple examination of the effect of increased uncertainty on equilibrium prices and building activity. Up to this point, I have not addressed issues relating to the effect of uncertainty on the current prices and rental rates of building units. In order to do this, I must add structure to the model. The following analysis examines a simple economy that consists of N identical lots that are initially vacant. If, in equilibrium, all the lots are developed at date 0, then there will exist no vacant lots to value. Conversely, if none of the lots are developed, no building units will exist. Hence, it makes sense to restrict the analysis to equilibria in which some, but not all, of the lots are developed at date 0. This suggests that, in equilibrium, the date 0 value of a vacant lot must equal the profit from developing it at that time:

$$V_0 = \Pi(p_0). \tag{19}$$

The demand for building units at date 0 is expressed as a decreasing function of their rental rate:

$$Q = nq^* = f(R_t), \tag{20}$$

where Q, the number of building units demanded, is equal to the product of n, the number of lots that are developed in the current period, and q^*, the number of building units constructed per lot. The function $f(R_t)$ is assumed to be continuous and differentiable with df/dR_t less than zero.

Equations (1), (2), (4), (8), (19), and (20), along with the exogenous p_l, p_h, and R_f, define a well-specified equilibrium.[5] The effect of uncertainty on this equilibrium can be explored in the manner developed in the previous section; by increasing p_h by x and decreasing p_l by xs_h/s_l so that $p_0 - (R_t/(1+R_f))$, s_h and s_l remain unchanged.

As was shown previously, an increase in uncertainty of this type leads to an increase in V. This implies that $\Pi(p_0)$ must increase, which in turn implies that both p_0 and q^* must increase. Since $p_0 - (R_t/(1+R_f))$ remains constant with changes in x, R_t must also increase. From equation (20) we see that Q decreases with increases in R_t. Since q^* increases and Q decreases, it must be the case that n decreases. In other words, if uncertainty is increased in a manner that keeps the state prices constant, prices of both land and building units as well as rental rates will increase, a larger portion of the land will remain vacant, but taller buildings will be constructed.

VI. Extensions and Practical Applications

Because of tractability considerations, the valuation model developed in Section II was kept simple. The model consisted of only two dates, with only two possible states of nature at the second date, and construction costs were assumed to be fixed. While these assumptions allow us to easily analyze the effects of uncertainty on land prices, they can be relaxed if our only interest is in developing a practical technique for valuing urban land.

[5]Note that the above equations are all continuous and that the variables are all finite and nonnegative. Hence, the existence of this equilibrium follows directly from Brouwer's fixed-point theorem.

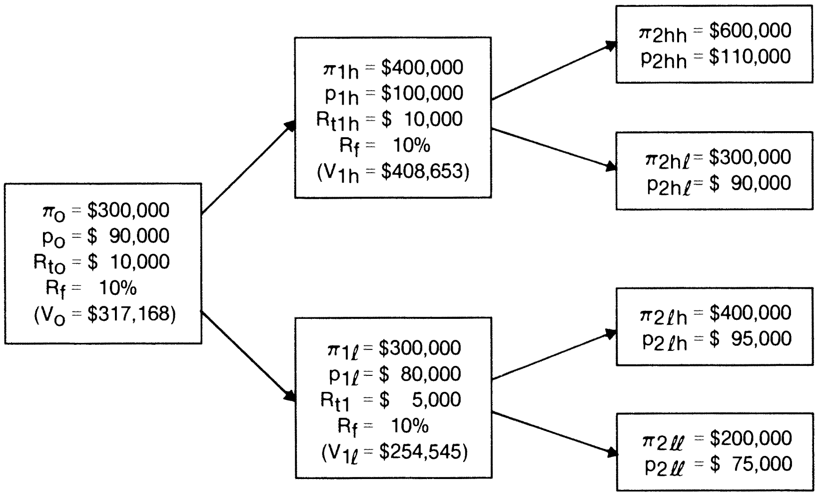

FIGURE 1

The assumption that construction costs are certain can easily be relaxed. The profit from constructing the optimal size building in each date and state of nature can be calculated as long as the construction costs and the per unit price of buildings is specified for each date and state. Substituting these profit levels into equation (8) yields the value of the vacant land.

The pricing model can also be generalized to allow for more than two dates. This can be done by specifying that for each date t state of nature, two possible date $t+1$ states of nature can occur. The date 0 land value can then be solved by backwards induction. For each state of nature at the second to last date, the vacant land value is given by equation (8). The larger of this value and the profit from developing the land in each state of nature at this date can then be substituted for Π into equation (8) to calculate the values of vacant land at the third to last date for the different states of nature. By continuing this process, we not only obtain the current value of the vacant land, but also determine at which future dates and states of nature the land is developed. Note also that by making the time periods between dates arbitrarily small and the number of dates arbitrarily large, we can have an arbitrarily large number of states of nature for each future time period. Hence, the assumption of only two date $t+1$ future states of nature for each date t state is not really restrictive.

The following numerical example illustrates this valuation method. It assumes three dates. The profit from developing the land, the per unit building price, and the rental rate is given for each date and state of nature in Figure 1. The value of the vacant land in the two date 1 states of nature are calculated in the manner specified in Section II. Since the value of the vacant land in the favorable state of nature ($408,635) exceeds the profits from developing the land in this state of nature, the land will remain vacant. However, the value of the land is only $254,545 in the unfavorable date 1 state of nature. Since this value is less than the profit from developing the land at that date, the land will be developed if the unfavorable state of nature occurs. Substituting the larger of the value of the vacant land and the profit from developing the land in each state of nature for $\Pi(p)$ in equation (8) yields the date 0 value of the vacant land. Since this value ($317,168) exceeds the profit from developing the land at date 0, the land will remain vacant at this date.

VII. Conclusion

The model developed in this paper provides a valuation equation for pricing vacant lots in urban areas. The analysis demonstrates that the range of possible building sizes provides a valuable option to the owner of vacant land that becomes more valuable as uncertainty about future prices increases. An implication of this relationship between uncertainty and vacant land values is that increased uncertainty leads to a decrease in building activity in the current period.

The relationship between building activity and uncertainty may have important macro implications. An article by Lawrence Summers (1981) and my 1982 article suggest that an increase in anticipated inflation leads to an increase in housing prices, which in turn leads to an increase in construction activity. The analysis presented here suggests that if the government initiates a monetary policy (or any other policy) to stimulate building activity, the policy may actually lead to a decrease in building activity if there is uncertainty about its duration or its effect.

The model also provides insights into the role of real estate speculators who purchase vacant lots, and rather than develop them immediately, choose to keep them vacant for a period of time. By waiting until some future date to build, the speculator is able to construct a building that is most appropriate given economic conditions at that time. Since the exact nature of these economic conditions are unknown at earlier dates, a building constructed earlier will not in general be the optimal size for the future. The decision to build or not build can thus be thought of as weighing the opportunity costs associated with keeping the land vacant against the expected gain from constructing a more appropriate building in the future.

It should also be noted that the framework developed here can easily be extended to analyze other issues relating to real estate pricing under uncertainty. For example, the analysis can easily be augmented to determine the value of houses that may or may not be torn down in the future so that the land can be used to develop large condominium complexes. The framework can also be used to determine when it is optimal to demolish a small building for the purpose of constructing a larger building, and under what conditions it is optimal to renovate an apartment house or convert it to condominiums. One could also use similar techniques to analyze the effect of uncertainty on the optimal durability of buildings.

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