Market Structure, Pigouvian Taxation, and Welfare

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Using a short-run partial equilibrium model of social welfare, this paper examines the social welfare implications of changing Pigouvian taxes under three markets: perfect competition, monopoly, and Cournot oligopoly. The result for perfect competition supports the earlier finding that Pigouvian taxation increases social welfare [Buchanan, 1969]. However, in contrast to the previous result that Pigouvian taxes lower welfare under monopoly, the authors show that if the noncompetitive distortion is small, these taxes might still be useful in correcting monopoly-generated externalities and in improving social welfare. Cournot firms react to the tax depending upon their individual perceptions of the gain in post-tax marginal revenue. Policy implications of the study's results are discussed. (JEL D6, D4)

Introduction

Economists have traditionally considered Pigouvian taxes (per-unit taxes) to be a feasible instrument for controlling externality-generating activities and restoring social efficiency.[1] There are, however, doubts in the literature about the efficacy of such a tax, especially when different market structures are considered.[2] From the Pareto-efficiency perspective, Buchanan [1969] graphically demonstrates that levying a per-unit corrective tax under external diseconomies may, contrary to the general belief, decrease social welfare when the externality-emanating industry is monopolistic. Only when the industry is competitive will such a tax be unambiguously welfare-improving. Recently, Martin [1986] questions the findings of Buchanan and shows that a Pigouvian tax might well increase social welfare under monopoly.

Understanding the welfare implications of Pigouvian taxation under different market structures has important public policy implications, especially in view of the growing attention being paid to pollution control. Using a unified neoclassical measure of social welfare, this paper examines the social welfare effects of changing Pigouvian taxes to control external discomfort from production under three well-known market structures: perfect competition, monopoly, and Cournot oligopoly. The paper also analytically investigates the effectiveness of Pigouvian taxation under monopoly [Buchanan, 1969]. While in general the results support the findings in the literature on Pigouvian taxation, a unique case under monopoly is found to be welfare-improving. This finding renders an

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alternative view on the effectiveness of the traditional corrective tax. The effects of changing Pigouvian taxes on Cournot markets are also discussed.

The Social Loss Function

A noncompetitive industry that creates some discomfort (like pollution) in its production process actually imposes two types of external diseconomies. One is the discomfort itself. The other is the deadweight loss due to underproduction which holds down output and forces consumers to face prices that are too high. Both are losses to the society and give rise to the social loss function (*SLF*), defined below. One might think of the discomfort as being pollution from oil refining.

In the short-run model there is a fixed number of $n \geq 1$ active firms in an industry. Each firm, indexed by i, $i=1, ..., n$, maximizes profit by producing a positive quantity, $x_i > 0$. Output of all firms is assumed to be homogeneous and is sold at the same price set by the inverse market demand function $P(X)$, where $X = \Sigma_i x_i$ is the level of industry output. It is also assumed that $P(X)$ is twice-differentiable, that $P(X) \geq 0$, and that the first derivative $P'(X) < 0$. Each firm incurs a total production cost $C_i(x_i) \geq 0$ and marginal cost c_i is taken to be a positive constant.

Production of each firm also imposes additional external damages on the society. Such "bads" are assumed to be directly related to the number of units produced and are invariant along an isoquant. External damage, for instance, may be interpreted as the effect of pollution on nonpolluting industries. Total social damage is assumed to be additively separable (i.e., each firm's damage is independent of other firms' damage) so that one can define each firm's external damage function as $Z_i(x_i) > 0$ and the marginal damage function as $z_i(x_i) > 0$. Total external damage is, therefore, equal to $\Sigma_i Z_i(x_i)$.

In order to examine the social welfare performance of a market equilibrium, especially within the noncompetitive framework, a hypothetical reference equilibrium is needed as a benchmark for comparison. Dixit and Stern [1982] provide a nice construct for such a purpose. The reference standard is built on the usual notions of production efficiency (i.e., least-cost method of production) and marginal cost pricing. Suppose that the product can be bought from the cheapest source at marginal cost c^* where:

$$c^* = \min_i \{c_i\} \tag{1}$$

and is equal to the price. In the presence of a per-unit Pigouvian tax t, the cheapest marginal cost becomes:

$$c^* + t = \min_i \{c_i + t\} . \tag{1'}$$

Since the main interest here is in examining the effects of changes in Pigouvian taxes, (1′) will serve as the benchmark in evaluating social welfare consequences. Given the market demand $X^*(t)$ corresponding to $(c^* + t)$, the consumer surplus can be evaluated as:

$$\int_0^{X^*(t)} P(q)dq \; - (c^* + t)\, X^*(t) \; . \tag{2}$$

According to (1′) and (2), it is possible for the society to have only one firm so long as that firm is the least-cost producer.[3] Also, there may be $n > 1$ identical firms. However, due to the noncompetitive distortion, the actual market price is higher than $(c^* + t)$ and a smaller consumer surplus is correspondingly created. This shrunken consumer surplus can be denoted as:

$$\int_0^{X(t)'} P(q)dq \; - P(X(t))\, X(t) \; , \tag{3}$$

which is based on the actual level of industry output, $X(t)$. Thus, the loss of consumer surplus from underproduction is equal to [(2) - (3)].

Producers' gain in the industry is the profit that they earn from operation. It can be expressed as:[4]

$$\Sigma_i \; [P(X(t)) \; - (c_i + t)]\, x_i(t) \; . \tag{4}$$

Given these assumptions, the deadweight loss to the society from underproduction relative to (tax-inclusive) marginal cost pricing is equal to the consumer surplus at $P = MC(t)$ minus the sum of the shrunken consumer surplus at $P > MC(t)$ and the producer profit [i.e., (2) - (3) - (4)].[5]

The output variables X, X^*, and x_i are determined in response to the respective post-tax prices, hence the notation $X(t)$, $X^*(t)$, and $x_i(t)$, respectively. Finally, the *SLF* is defined as the sum of the deadweight loss to the society, [(2) - (3) - (4)], and the total external damage, $\Sigma_i\, Z_i(x_i(t))$. Formally, after rearranging,[6]

$$\begin{aligned} SLF \; = \; & \int_0^{X^*(t)} P(q)dq \; - (c^* + t)\, X^*(t) \; - \; \int_0^{X(t)} P(q)dq \\ & + \Sigma_i \; (c_i + t)\, x_i(t) \; + \Sigma_i Z_i(x_i(t)) \; . \end{aligned} \tag{5}$$

The changes in social welfare as a result of changes in the Pigouvian tax are discussed in the next section.

Social Welfare Consequences of a Pigouvian Tax

Diagnosing the social welfare effect of changes in a Pigouvian tax to control external discomfort from production is equivalent to determining the sign of $(dSLF/dt)$ under a particular market structure. A negative $(dSLF/dt)$ implies an increase in social welfare due to the tax change and a positive $(dSLF/dt)$ indicates otherwise.

Differentiating (5) with respect to t yields:[7,8]

$$(dSLF/dt) = X(t) - X^*(t) - \Sigma_i [P(X(t)) - (c_i + t)] (dx_i(t)/dt) + \Sigma_i z_i(x_i(t))(dx_i(t)/dt) , \quad (6)$$

where $X(t)$ and $X^*(t)$ are evaluated using $d(c_i + t)/dt \equiv 1$ and $d(c^* + t)/dt \equiv 1$, respectively.[9] The last term in (6) shows the effect of a change in the Pigouvian tax on external damage, while the other terms comprise the effect on the deadweight loss due to underproduction. Next, the welfare effects of Pigouvian taxation under the three market structures are considered.

Perfect Competition

When the industry is perfectly competitive, there is no noncompetitive distortion. In this study's framework, this implies that $X(t) = X^*(t)$ and $P(X(t)) = c_i + t = c^* + t$ for all i.[10] Thus, (6) reduces to:

$$(dSLF/dt) = \Sigma_i z_i(x_i)(dx_i/dt) , \quad (6')$$

so that the change in social loss (welfare) is solely determined by the change in the external damage from production. In order to proceed with the analysis one needs to determine the impact of a tax change on firm output, i.e., the sign of (dx_i/dt) in (6′).

A profit-maximizing competitive firm produces output to the point where:

$$P(X) = c_i + t, \quad \text{for all } i . \quad (7)$$

Then $(dX/dt) = (dx_i/dt) = 1/P'$,[11] which is unambiguously negative by assumption and so is the sign of $(dSLF/dt)$ for a competitive industry. This implies that each competitive firm responds to the higher after-tax marginal cost by lowering its output and, thus, emits a diminished external discomfort. Society as a whole is, therefore, made better-off.[12] This result is consistent with earlier findings in the literature [Buchanan, 1969] and justifies the traditional popularity of such a tax regime.

Monopoly

To make the analysis more general, allow for the monopolist's marginal costs $(c + t)$ to be different from the cheapest marginal cost $(c^* + t)$. One reason for this difference might be monopolistic X-inefficiency. In a monopoly, by definition, $i = 1$ and $x_i(t) = X(t)$ and (6) becomes:

$$(dSLF/dt) = X - X^* - [P(X) - (c + t)](dX/dt) + z(X)(dX/dt) . \qquad (6'')$$

It can be seen in (6″) that the pattern of the change in social loss is more complicated than that shown in (6′). Under monopoly, the effect of a change in the Pigouvian tax on both the external damage and the deadweight loss comes into play. Assume that $P' < 0$ and stability requires that $[P' + P''X] < 0$ [Hahn, 1962].[13] Post-tax monopoly equilibrium (i.e., $MR = MC$) gives:

$$P + P'X - c - t = 0 \qquad (8)$$

and

$$(dX/dt) = 1/[2P' + P''X] , \qquad (9)$$

which is unambiguously negative.

Equation (9) implies that the monopolist decreases output when faced with a higher tax. While the expected level of social damage from production will be smaller than before, the magnitude of the already existing monopolistic distortion from underproduction will increase. From (6″), these two effects imply that $[z(X)(dX/dt)] < 0$ and $\{X - X^* - [P(X) - (c + t)][dX/dt]\} > 0$. Given the fact that $X < X^*$, the necessary condition for the latter to be true is that $[P(X) - c - t]$ is positive, i.e., the product price is higher than the tax-inclusive marginal production cost. This condition is certainly justified by the noncompetitive market theory. It can be seen that $(dSLF/dt)$ in (6″) can be positive, indicating that the social loss (welfare) is increased (reduced) by a Pigouvian tax, when the decrease in external damage is more than offset by the increase in noncompetitive distortion. However, it can be true only if $\{[P(X) - c - t](dX/dt) - [X - X^*]\} << 0$, i.e., $[P(X) - c - t](dX/dt)$ is much less than $[X - X^*]$. Since $[X - X^*]$ is negative, this implies that $[P(X) - c - t]$ must be very positive (since $(dX/dt) < 0$), suggesting a more severe distortion in resource allocation.[14] It is clear from the derivation that if the price net of costs and taxes, $[P(X) - c - t]$, is positive but small in magnitude, $(dSLF/dt)$ can be negative. In such a case, Pigouvian taxation would lead to improved social welfare. Therefore, taxation of a monopoly may be socially beneficial.

Cournot Oligopoly

If a Cournot oligopoly prevails, the derivative (6) is relevant since zero conjectural variations, $(dx_i/dx_j) = 0$, $i \neq j$, have been imposed in deriving (6). The format of the adjustments following a tax change is quite similar to that under monopoly, (6″), except under Cournot oligopoly more sophisticated behavior appears due to interaction among firms and the result is less straightforward. In an oligopoly, $[P(X) - c_i - t]$ for each producer is again positive and $[X - X^*]$ is negative.

To analyze Cournot oligopoly, a methodology similar to Levin [1985] is used. Three alternate Cournot situations are considered (see the Appendix for details). First, in Case A, if all firms in the oligopoly are of equal size and produce equal output, then (dx_i/dt) is unambiguously negative for all i (see equation (A6) in the Appendix). The social welfare diagnosis under this special situation is similar to that of a monopoly discussed above. While the external discomfort obviously decreases, social welfare may decrease or increase, depending upon the magnitudes of $[P(X) - c_i - t]$ for all firms.

In Case B, if the demand curve is sharply concave $(P'' << 0)$, implying that the elasticity of demand quickly becomes quite small at large outputs, then firms with higher-than-average pre-tax output levels will increase production, while smaller firms will produce less. Such an unusual phenomenon of a simultaneous existence of increasing and decreasing firm outputs following a change in Pigouvian tax might be the result of asymmetric marginal revenue effects on firms of different size (see Levin [1985] for support of this argument). It implies that by raising production, large firms can benefit from higher after-tax marginal revenues.[15] In other words, with asymmetric firms and symmetric Pigouvian taxes the heavy polluters will benefit. Obviously, the net effect on pollution cannot be determined without detailed information on large and small firms.

In the second part of Case B, the situation where the demand curve is only mildly concave $(P'' < 0)$, it is shown that smaller firms will again decrease production but large firms may go either way, depending upon the relative magnitude of the advantage from higher after-tax marginal revenues. It is clear from (6) that the sign of $(dSLF/dt)$ cannot be determined without detailed information on not only the direction and size of each firm's adjustment in output, but also the magnitudes of the resulting changes in the external discomfort and the noncompetitive distortion. Here again the magnitudes of $[P(X) - c_i - t]$ will dictate the change in distortion and, hence, affect the change in social welfare.

Case C assumes the opposite situation, where the demand curve is taken to be convex $(P'' >> 0$ or $P'' > 0)$, implying that demand is more elastic at higher output levels. Compared to Case B, here the arguments about firms' output adjustments are just the reverse. Social welfare implications are again ambiguous. For example, larger firms will reduce output while smaller firms will increase output when $P'' >> 0$. The same kinds of information mentioned in Case B are necessary to draw more definite conclusions.[16] The results under the three market structures are summarized in Table 1.

Conclusion

Using a short-run partial equilibrium model of social welfare, this study examines the validity of changing Pigouvian taxes under three markets: perfect competition, monopoly, and Cournot oligopoly. The result for perfect competition supports the earlier finding that higher Pigouvian taxes increase social welfare [Buchanan, 1969]. However, in contrast to the traditional result that higher Pigouvian taxes lower welfare under monopoly, the study shows that if the noncompetitive distortion is small, these taxes might still be a useful tool in correcting monopoly-generated externalities and in improving social welfare. The same argument also has bearings on the welfare implications of a Pigouvian tax under Cournot oligopoly. In such a market, firms generally react to the tax depending upon their individual perceptions of the gain in after-tax marginal revenue. Some oligopolists will increase output while others might choose to produce less. The informational requirements for welfare diagnosis in this case are quite demanding.[17] To evaluate welfare impacts, one must have detailed information about the direction and size of each Cournot firm's post-tax output adjustment, the magnitude of the resulting change in external damage, as well as the change in the degree of noncompetitive distortion.

TABLE 1
Welfare Impacts of Pigouvian Taxes

Market Structure	Effect on Welfare	Condition
1. Perfect Competition	+	$P' < 0$
2. Monopoly		
(i)	-	$(P - c - t) >> 0$
(ii)	+	$(P - c - t) > 0$
3. Cournot Oligopoly		
3a. Identical Firms		
(i)	-	$(P - c_i - t) >> 0$
(ii)	+	$(P - c_i - t) > 0$
3b. Concave Demand Curve		
(i) $P'' << 0$	+/-	Need information on small and large firms
(ii) $P'' < 0$	?	Need more information
3c. Convex Demand Curve		
(i) $P'' >> 0$	+/-	Need information on small and large firms
(ii) $P'' > 0$	?	Need more information

One public policy implication of this analysis is that the effectiveness of Pigouvian taxation is dependent on the market structure. Therefore, a blanket pollution control tax change across industries is likely to produce different welfare effects across different industries. For example, while greater taxation of an externality-producing competitive industry is socially beneficial, taxation of monopoly and Cournot markets might turn out to be either welfare-enhancing or welfare-decreasing.

APPENDIX
Cournot Oligopoly

A Cournot firm maximizes net profits by choosing output x_i:

$$\max \quad \Pi_i = P(X)x_i - (c_i + t)x_i \; .$$

The marginal conditions for profit maximization are:

$$P + P'x_i - c_i - t = 0, \quad \text{for all } i \; . \tag{A1}$$

Differentiating (A1) with respect to t gives:

$$(dx_i/dt) = [1 - (P' + P''x_i)(dX/dt)]/P' \tag{A2}$$

Summing over all i and rearranging terms, one obtains:

$$(dX/dt) = n/[P' + (nP' + P''X)] \; . \tag{A3}$$

It has been assumed that $P' < 0$ and stability requires that $[P' + P''x_i] < 0$ [Hahn, 1962]. Therefore, total output is unambiguously reduced after the tax.

Substituting (A3) into (A2) gives:

$$(dx_i/dt) = [P' + P''(X - nx_i)]/[P'((1+n)P' + P''X)] \; . \tag{A4}$$

The denominator on the right-hand-side of (A4) is positive by assumed stability and negative slope of demand. Therefore:

$$(dx_i/dt) \gtreqless 0, \quad \text{as} \quad P' \gtreqless P''(nx_i - X) \; . \tag{A5}$$

Three cases are possible:

$$\text{(a)} \quad \text{If } x_i = X/n, \; (dx_i/dt) < 0 \; . \tag{A6}$$

(b) If $P'' \ll 0$, $(dx_i/dt) > 0$ iff $x_i > X/n$, and

$(dx_i/dt) < 0$ iff $x_i < X/n$.

If $P'' < 0$, $(dx_i/dt) < 0$ iff $x_i < X/n$, and

(dx_i/dt) ? iff $x_i > X/n$.

(c) If $P'' \gg 0$, $(dx_i/dt) > 0$ iff $x_i < X/n$, and

$(dx_i/dt) < 0$ iff $x_i > X/n$.

If $P'' > 0$, $(dx_i/dt) < 0$ iff $x_i > X/n$, and

(dx_i/dt) ? iff $x_i < X/n$.

Footnotes

1. See Lin [1976] for a comprehensive treatment of economic externalities.
2. For example, Carlton and Loury [1980] point out that, even in a competitive industry, a Pigouvian tax alone will not, under certain conditions, lead to a social optimum in the long run. Polinsky and Shavell [1982] argue that when administrative costs are present in enforcing such a tax, the tax should be adjusted to reflect these costs. Hahn [1989] discusses the superiority of marketable permits over effluent charges. Further, Levin [1985] shows that under Cournot oligopoly a Pigouvian tax might increase, rather than decrease, pollution due to cost-function asymmetries. Also see Oates and Strassmann [1984].
3. With constant marginal costs and no fixed costs, the authors are ruling out a natural monopoly situation.
4. Note that the formulation used here allows for the possibility of nonidentical costs, i.e., $c_i \neq c_j$, for $i \neq j$.
5. Here the income distribution effects within and among different groups are ignored.
6. This paper does not intend to describe optimal Pigouvian tax rules (see Barnett [1980] for a model of optimal Pigouvian taxes under monopoly). The present exercise aims at examining the social welfare responses to a tax change. Hence, in the post-tax scheme, the authors again assume that market equilibrium is attained. Also see footnote 16.
7. Use $P(X^*(t)) = c^* + t$ for the benchmark case, i.e., the case of marginal cost pricing.
8. Here the authors abstract from terms embodying firm interdependence, i.e., $dx_i/dx_j = 0$, for $i \neq j$.
9. With nonlinear costs, on the other hand, additional cost terms would appear in (6), (6′), and (6″). Consequently, the welfare rankings under different market structures would be less clear-cut.
10. For notational ease, X and x_i will henceforth imply $X(t)$ and $x_i(t)$, respectively.
11. Using $P(X) = c_i + t$, then $(dP/dX)(dX/dt) = 1$. Consequently, $(dX/dt) = (1/P')$.
12. One should note that the analysis here is partial equilibrium. Social welfare might diminish if other industries increase their external damage due to the tax. Furthermore, the results are subject to the initial level of the tax. If a significant negative externality exists and the initial tax is zero, an increase in the tax might benefit the society. On the other hand, if the initial tax

is extremely large, lowering the tax has the same beneficial effect on welfare. These conclusions are also likely to hold in other nonextreme situations.

13. Hahn's [1962] condition can be interpreted as stating that the slope of the demand curve should be less than the slope of the marginal revenue curve. This is true for all price searchers facing negative sloping demands, including monopolists. If, on the other hand, the slope of the marginal revenue is less than the slope of the demand curve, then no profit-maximizing output will exist. In this sense, equilibrium under Hahn's condition is stable.
14. This condition about $[P(X) - c - t]$, which leads to a reduction in social welfare and so the failure of Pigouvian taxation, is stronger than the one in Buchanan [1969]. Buchanan [1969, p. 176] states that "the result $((dSLF/dt) > 0)$ must hold so long as the corrective tax, which we assume to have been estimated properly, is less than the difference between price and marginal revenue (marginal cost) at the initial monopoly output."
15. In the simple case of a linear demand (say $P(X) = a - bX$), stability is guaranteed and (dx_i/dt) will always be negative. The distribution of output among Cournot firms in this case will be irrelevant.
16. Assuming optimality, one could also derive the optimal Pigouvian tax rules, t^*, in different market environments. Setting (6) equal to zero, solving for t, and applying the same market conditions as previously imposed in deriving (6′) and (6″), the optimal tax t^* is equal to:

(i) $\Sigma_i z_i (x_i)(dx_i/dt) / [\Sigma_i (dx_i/dt)]$ under perfect competition ;

(ii) $[X - X^* - (P(X) - c)(dX/dt) + z(X)(dX/dt)] / (dX/dt)$ under monopoly

(iii) $[X - X^* - \Sigma_i (P(X) - c)(dx_i/dt) + \Sigma_i z_i(x_i)(dx_i/dt)] / \Sigma_i (dx_i/dt)$
under Cournot oligopoly .

It is clear from the three expressions that the same factors which influence the final welfare diagnoses discussed in this section are also the ones which determine the optimal tax levels, t^*. If the market is perfectly competitive, t^* depends only upon the marginal external damage and the individual firm's output responses. In a monopoly, the extent of the monopolistic distortion comes into play. Under Cournot oligopoly, the substantial informational requirements about a firm's output responses again lead to indeterminate results.

17. Hahn [1989], for instance, points out the difficulty in collecting information to evaluate the effect of emission charges. The informational requirements under oligopoly are also recognized by Katz and Rosen [1985].

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